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ASTU

School of Electrical Engineering and Computing

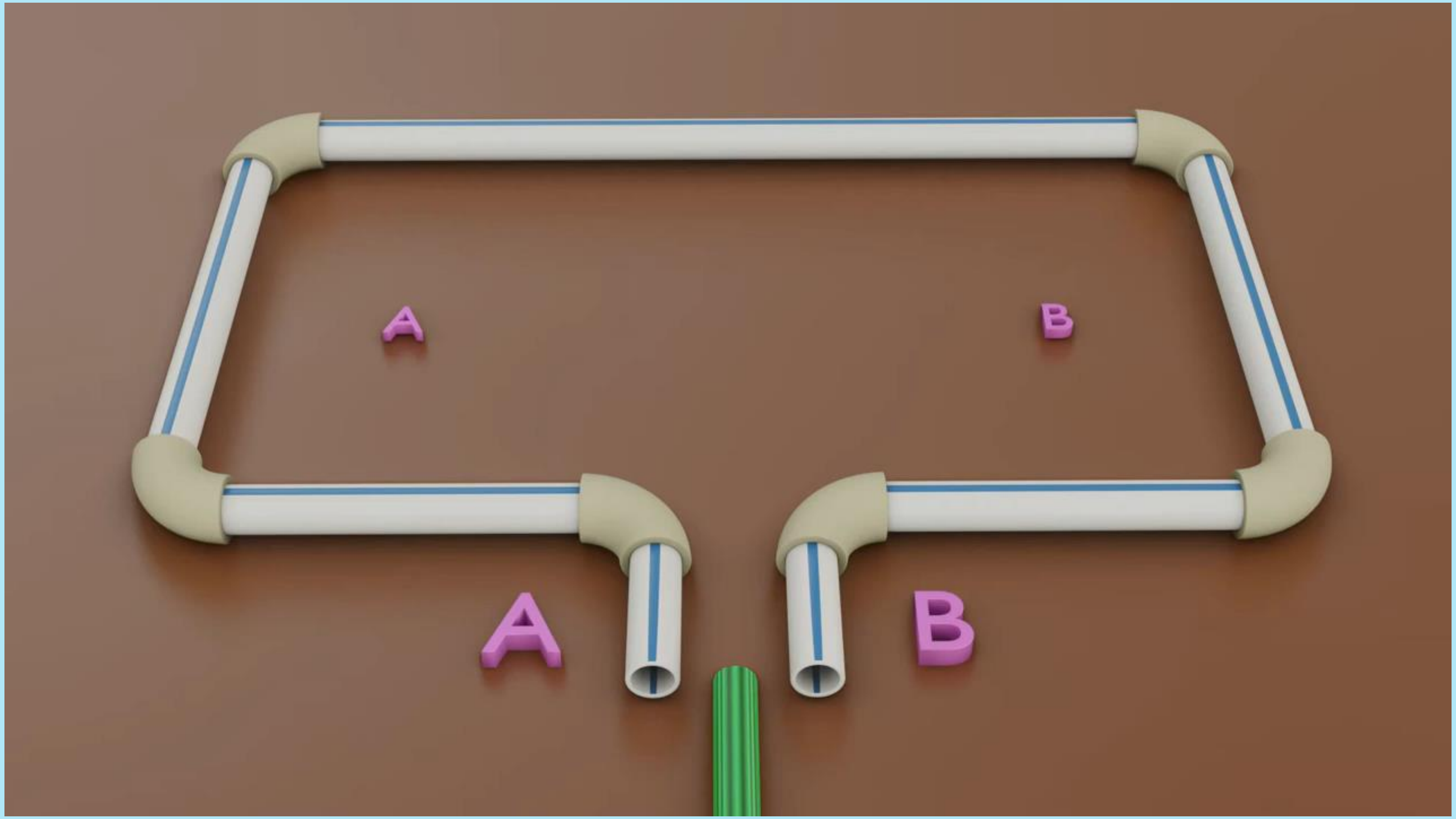
Department of Electrical Power and Control Engineering

Fundamentals of Electrical Engineering (EPCE 2101)

Chapter – 5
AC Steady State Analysis

Outlines

AC



5.1 Introduction to Sinusoids

- **A sinusoid** is a signal that has the form of the sine or cosine function.
- A sinusoidal current is usually referred to as **ac.**
- Such a current reverses at regular time intervals and has alternately positive and negative values.
- **Circuits driven by sinusoidal current or voltage sources are called ac circuits**.

5.2. Sinusoidal and complex forcing functions

- Sinusoids are easily expressed in ***terms of phasors***,

A phasor is a complex number that represents the amplitude and phase of a sinusoid.

- A **general expression for the sinusoid**,

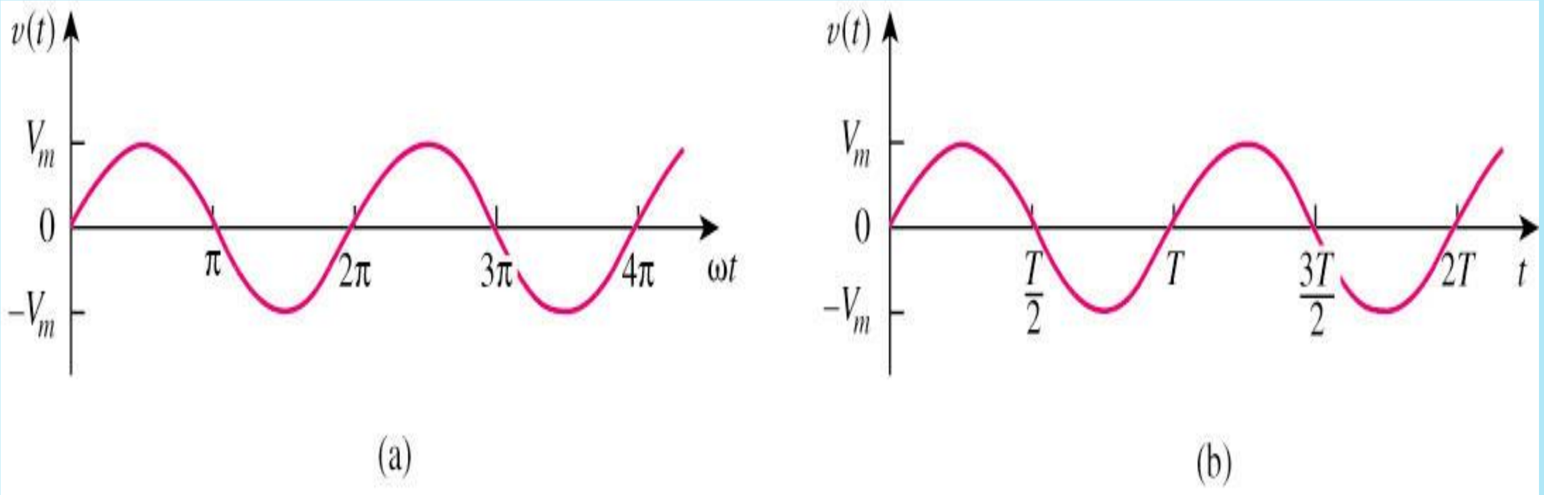
$$v(t) = V_m \sin(\omega t + \phi)$$

Where

V_m = **Amplitude** of the sinusoid
ω = **Angular frequency** in radians/s
Φ = **Phase**
ωt = **Argument of the sinusoid**

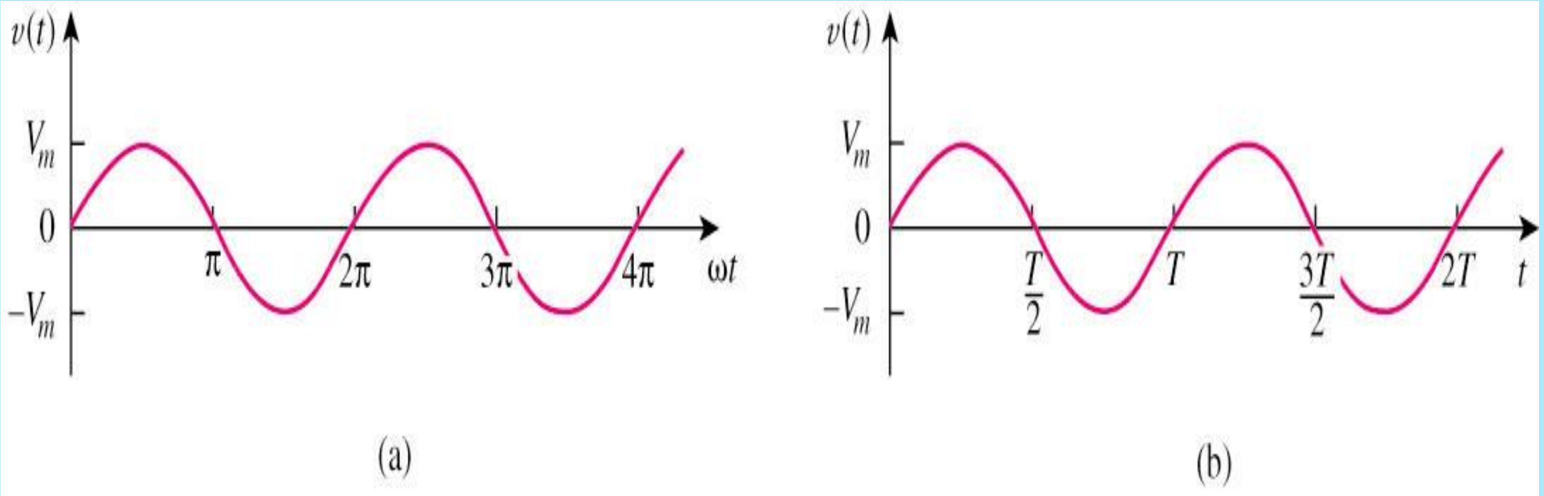
5.2. Sinusoidal and complex forcing functions

- The sinusoid is shown in
  - Fig. 5.1(a) as a ***function of its argument*** and
  - Fig. 5.1(b) as a ***function of time***.



A sketch of ***Vm* sin *ωt***: (a) as a function of ***ωt*,** (b) as a function of *t*.

5.2. Sinusoidal and complex forcing functions



A sketch of ***Vm* sin *ωt***: (a) as a function of ***ωt***, (b) as a function of *t*.

- The sinusoid repeats itself every T second; thus, **T is called the period of the sinusoid.**

$$\omega T = 2\pi, \rightarrow T = \frac{2\pi}{\omega} \qquad \omega = 2\pi f$$

5.2. Sinusoidal and complex forcing functions

- ***A periodic function is one that satisfies f (t) = f (t + nT), for all t and for all integers n.***

$$v(t+T) = V_m \sin\omega(t+\frac{2\pi}{\omega}) = V_m \sin\omega(t+\frac{2\pi}{\omega})$$

- The ***period T*** of the periodic function is the time of one complete cycle or the number of seconds per cycle.

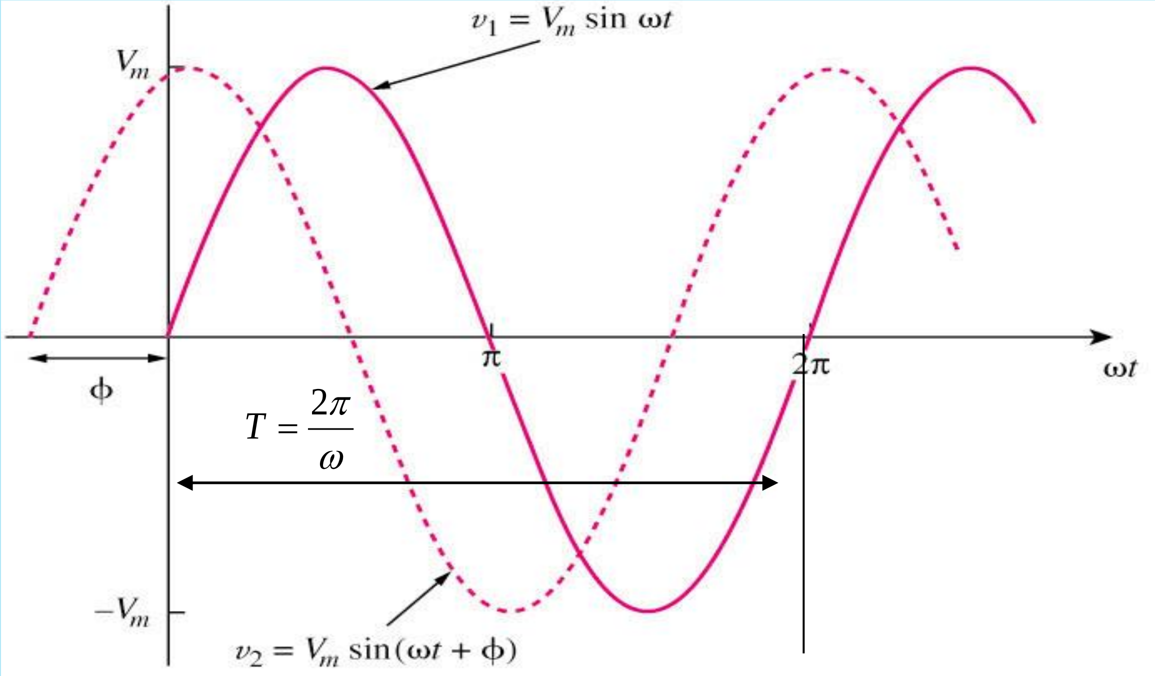
5.2. Sinusoidal and complex forcing functions

$$\omega = 2\pi f$$

$$f = \frac{1}{T}\ Hz$$

$v_1(t) = Vm \sin \omega t$ and $v_2(t) = Vm \sin(\omega t + \phi)$

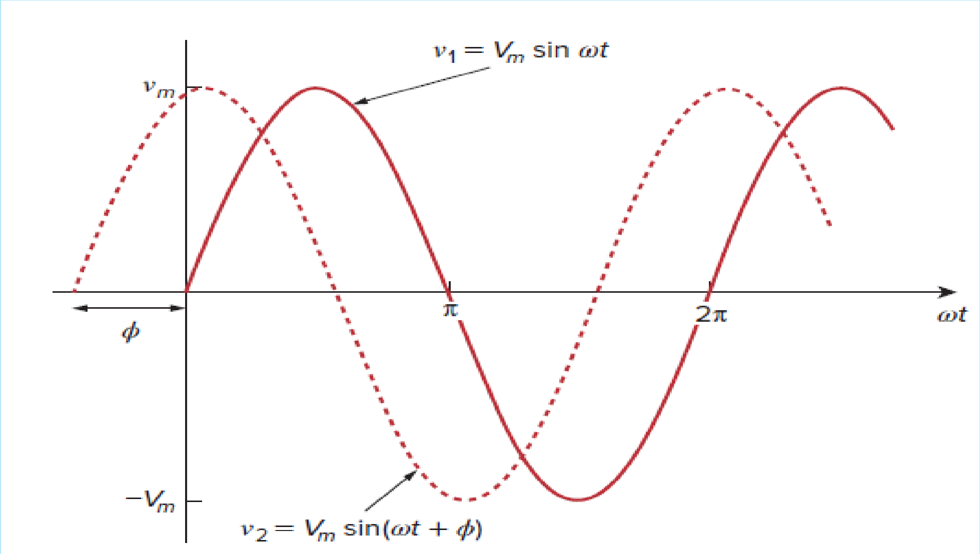
- ***v_2 leads v1 by ϕ or that v1 lags v2 by ϕ.***
- ***If $\phi \neq 0$, we also say that v_1 and v_2 are out of phase.***
- ***If $\phi = 0$, then v_1 and v_2 are said to be in phase.***



Two sinusoids with different phases.

5.2. Sinusoidal and complex forcing functions

- Their ***minima and maxima at exactly the same time***.
- We can compare v_1 and v_2 in this manner because ***they operate at the same frequency; they do not need to have the same amplitude***.



Two sinusoids with different phases.

5.2. Sinusoidal and complex forcing functions

- **A sinusoid** can be expressed in either sine or cosine form.
- When comparing two sinusoids, it is convenient to express both as either sine or cosine with positive amplitudes.

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$
$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

5.2. Sinusoidal and complex forcing functions

Using these relationships, we can **transform a sinusoid from sine form to cosine form or vice versa.**

$$\sin(\omega t \pm 180°) = -\sin \omega t$$

$$\cos(\omega t \pm 180°) = -\cos \omega t$$

$$\sin(\omega t \pm 90°) = \pm \cos \omega t$$

$$\cos(\omega t \pm 90°) = \mp \sin \omega t$$

$$\cos(\omega t - 90°) = \sin \omega t, \quad \sin(\omega t + 180°) = -\sin \omega t$$

5.2. Sinusoidal and complex forcing functions

We can add two sinusoids of the same frequency when

- One is in sine form and
- The other is in cosine form

To add **A cos ωt and B sin ωt**, we note that A is the magnitude of cos ωt while B is the magnitude of sin ωt,

$$A\cos\omega t + B\sin\omega t = C\cos(\omega t - \theta)$$

$$C = \sqrt{A^2 + B^2}$$

$$\theta = \tan^{-1}\frac{B}{A}$$

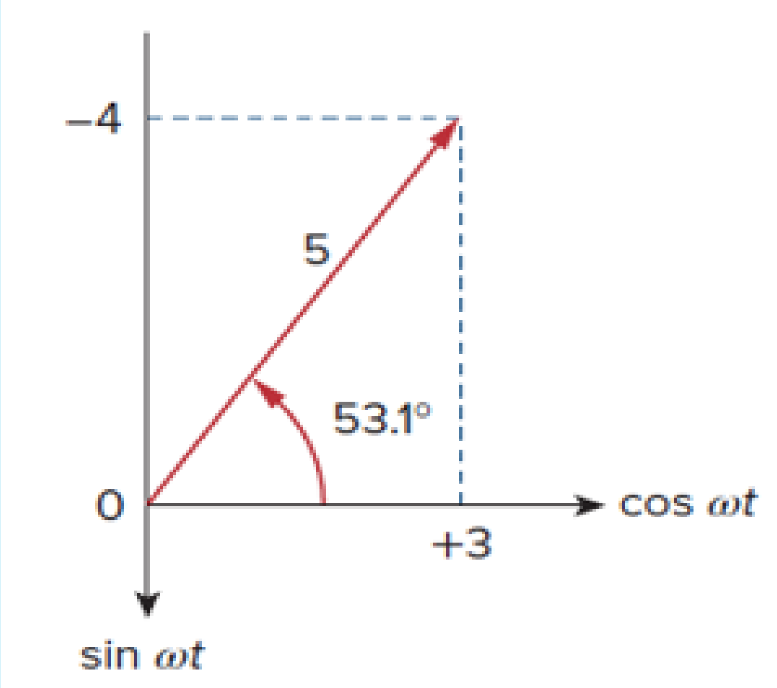
Example

1. Add 3 cos ωt and −4 sin ωt as shown in Fig. and obtain

$$\mathbf{3\cos \omega t - 4\sin \omega t = 5\cos(\omega t + 53.1^\circ)}$$

$$C = \sqrt{A^2 + B^2}$$

$$\theta = \tan^{-1}\frac{B}{A}$$



Example

1. Find the amplitude, phase, period, and frequency of the sinusoid

$$V(t) = 12\cos(50t + 10°)\ \text{V}.$$

Solution:

- The amplitude is $V_m = 12$ V.
- The phase is $\phi = 10°$.
- The angular frequency is $\omega = 50$ rad/s.
- The period $T = \frac{2\pi}{\omega} = \frac{2\pi}{50} = 0.1257$ s.
- The frequency is f = 1/T = 7.958 Hz.

Activity

1. Two sources have frequencies f_1 and f_2 respectively.

If $f_2 = 2f_1$ and T_2 is 20ms, determine f_1, f_2, and T_1?

$$f_2 = \frac{1}{T_2} = \frac{1}{20ms} = 50Hz$$

$$f_1 = \frac{f_2}{2} = \frac{50Hz}{2} = 25Hz$$

$$T_1 = \frac{1}{f_1} = \frac{1}{25Hz} = 40ms$$

Homework

1. Given the sinusoid **45 cos(5πt + 36°),** calculate its amplitude, phase, angular frequency, period, and frequency.

Solution:

- The amplitude is $V_m = 45$ V.
- The phase is $\phi = 36°$.
- The angular frequency is $\omega = 15.708$ rad/s.
- The period $T = \frac{2\pi}{\omega} = \frac{2\pi}{15.708} = 400$ms.
- The frequency is f = 1/T = 2.5Hz.

Example

1. Calculate the ***phase angle between*** $v_1 = -10\cos(\omega t + 50°)$ and $v_2 = 12\sin(\omega t - 10°)$. State which sinusoid is leading.

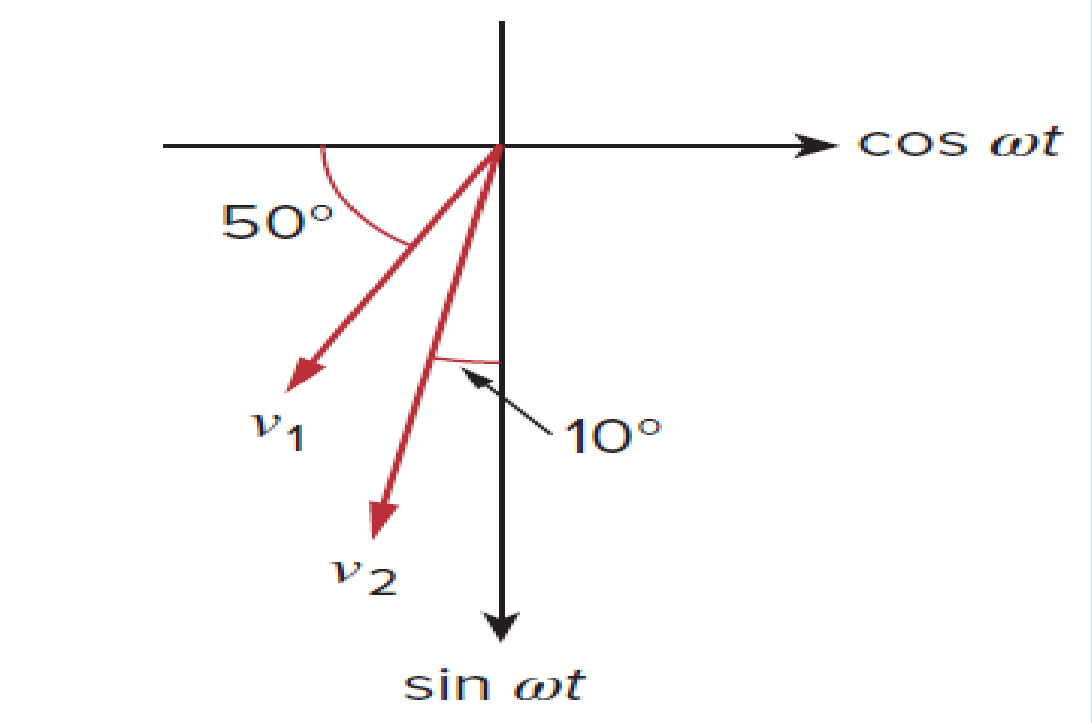
Solution In order to compare v_1 and v_2, ***we must express***

- ***In the same form.***
- ***In positive amplitudes,***

$$\cos\omega t = \sin(\omega t + 90°)$$
$$-\cos\omega t = \sin(\omega t - 90°)$$
$$\sin\omega t = \cos(\omega t - 90°)$$
$$-\sin\omega t = \cos(\omega t + 90°)$$

Example

Phase difference between v_1 and v_2 is 30°.



v_2 leads v_1 by 30°

Homework

Does i_1 lead or lag i_2?

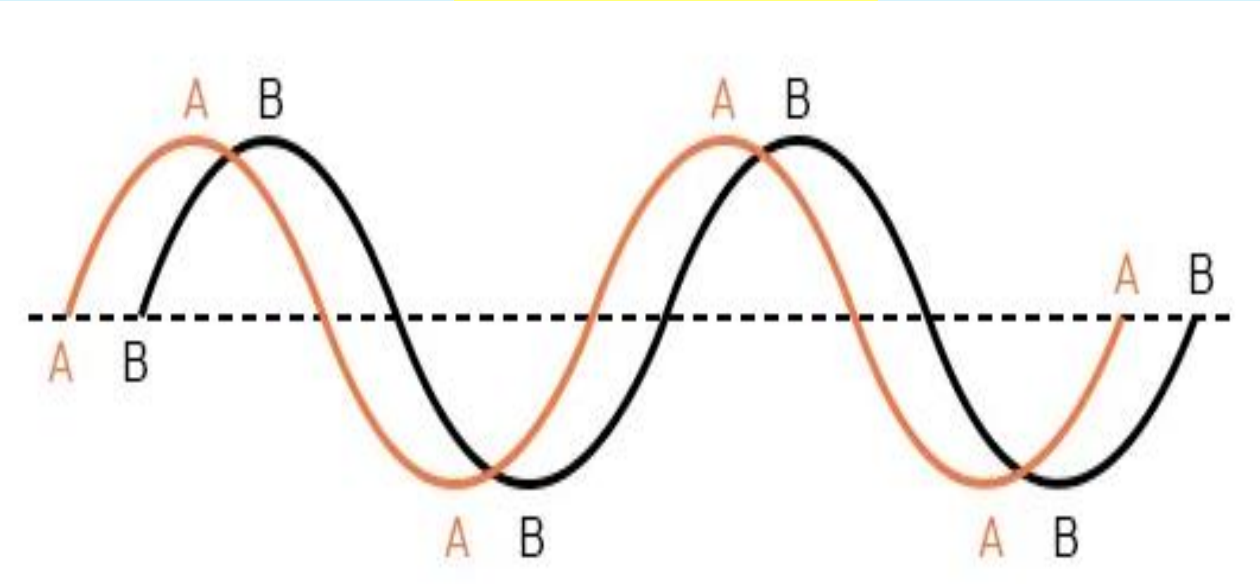
$$i_1 = -4\sin(377t + 55°) \text{ and } i_2 = 5\cos(377t - 65°)$$

210°, i_1 leads i_2.

AC Phase

1. Out of phase:

- By "out of step," I mean that the two waveforms **are not synchronized.**
- Their ***peaks and zero points*** do not match up at the same points in time



Out of phase waveforms.

Using a graphical approach to relate/compare sinusoids

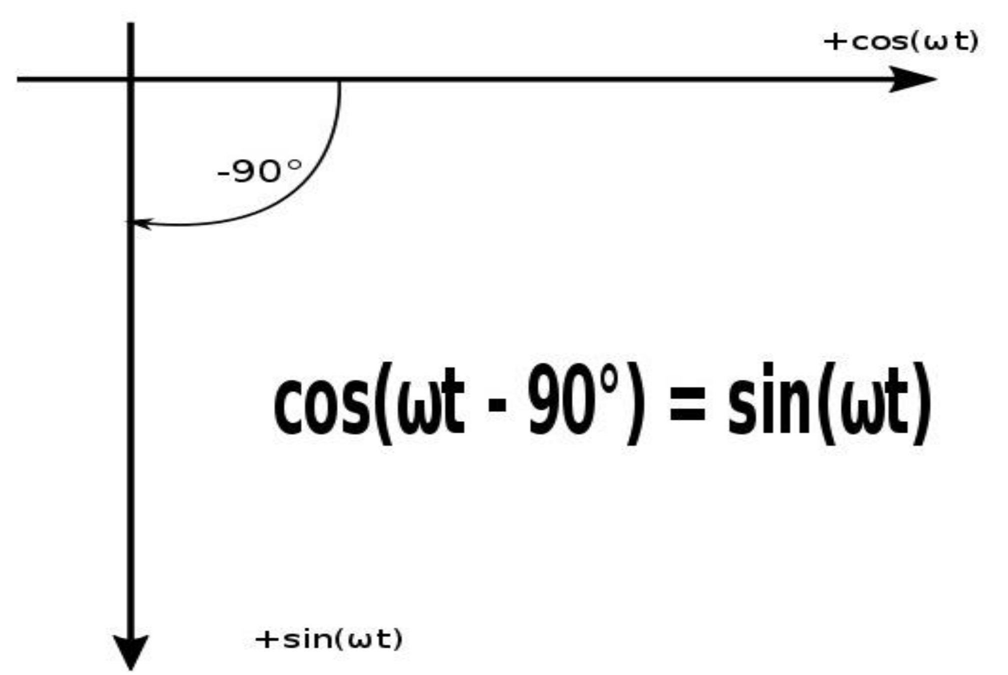
- We can also compare sinusoids that are expressed as **sines and cosines** with a **graphical approach**.

❖ **With this method,**

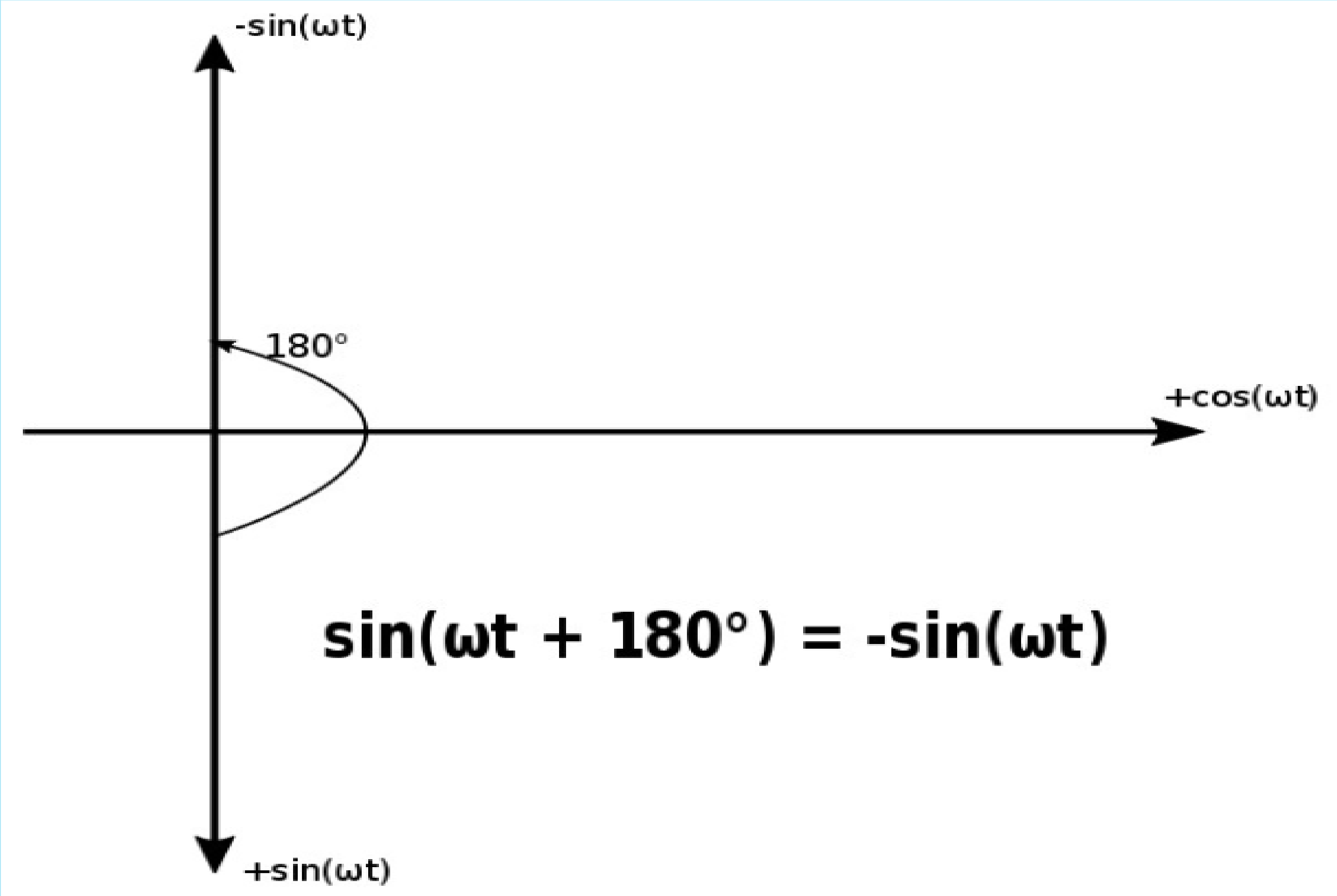
- The **horizontal axis represents the magnitude of cosine** and the **vertical axis represents the magnitude of sine**.

Using a graphical approach

- The positive direction of sine is denoted as pointing "downwards". **Angles are measured** positively in a counter-clockwise direction from the **horizontal axis**.



Another illustration of the graphical approach



Graphical Method to Add/Subtract Sinusoids of the Same Frequency

- Add **the following two sinusoids (v_1 and v_2):**

$$v_1(t) = Acos(\omega t)$$

$$v_2(t) = Bsin(\omega t)$$

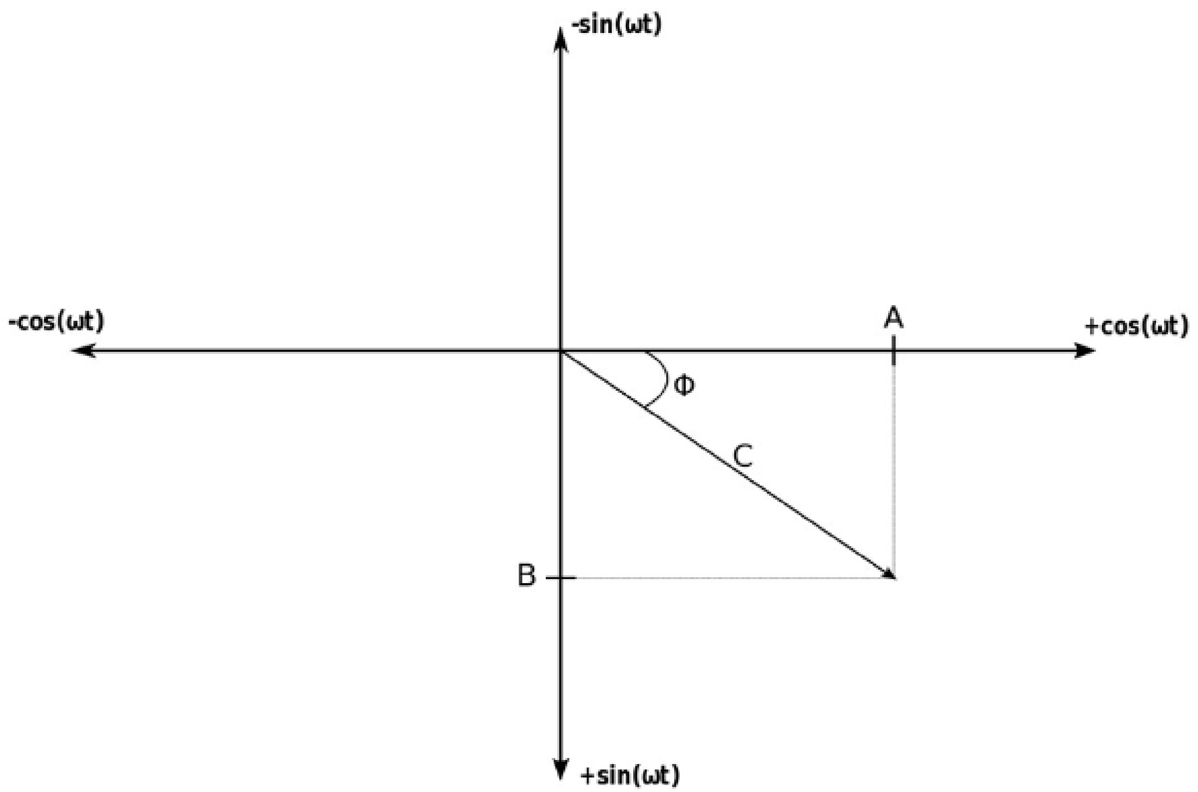
Graphical Method to Add/Subtract Sinusoids

- The magnitude and argument of the resulting sinusoid can be obtained from the following triangle:

$$v_1(t) = Acos(\omega t)$$

$$v_2(t) = Bsin(\omega t)$$

$$C = magnitude\ of\ resultant\ sinusoid$$



Graphical Method to Add/Subtract Sinusoids

- The phase angle can be determined by the definition of tangent

$$\tan(\phi) = \frac{B}{A}$$

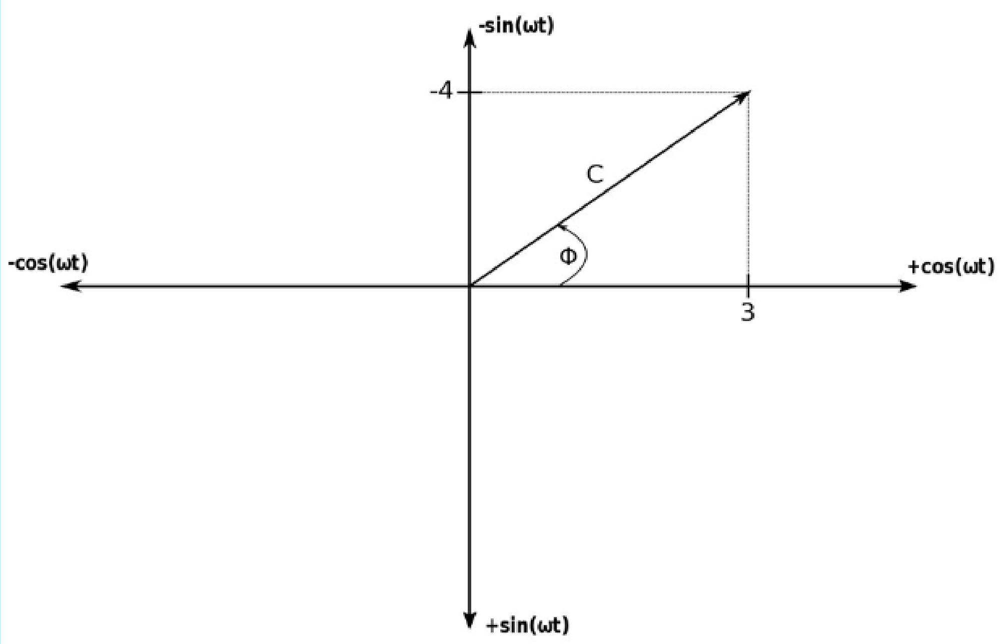
$$\phi = \tan^{-1}\left(\frac{B}{A}\right)$$

- For the phase angle, **don't forget** the difference between a reference angle (relative to the horizontal axis) and the actual angle.
- The sign of A and B will tell you what quadrant of the plane the **angle lies.**

Sum the following sinusoids

$$3cos(\omega t) - 4sin(\omega t)$$

✓ **Start by using the graphical approach to plot the sinusoids:**



Phasors (Sinusoid Example Problems)

- **Calculate the phase angle between**:

$$v_1 = -10cos(\omega t + 50^\circ)$$

$$v_2 = 12sin(\omega t - 10^\circ)$$

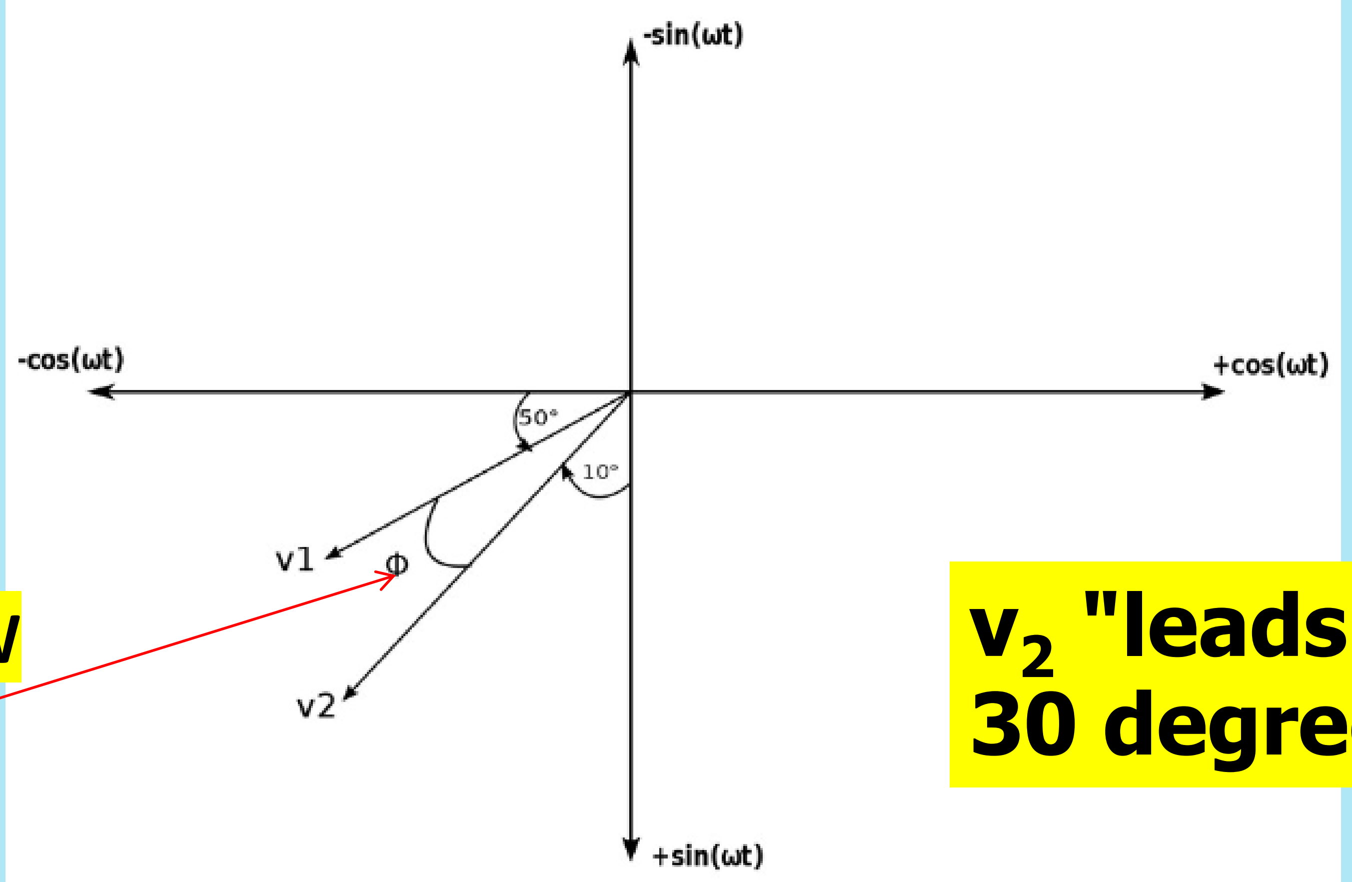
Using both the method of trig identities and the graphical approach. **Determine which sinusoid is leading.**

Method #2 (Graphical Approach)

- plotting v_1 and v_2 on our "improvised" coordinate system:

$$v_1 = -10cos(\omega t + 50^{o})$$

$$v_2 = 12sin(\omega t - 10^{o})$$



$$\phi = 30^{o}$$

HW

v_2 "leads" v_1 by 30 degrees.

Phasor

- **A phasor** is a complex number that represents the amplitude and phase of a sinusoid.
- Phasors provide a simple means of analyzing linear circuits excited by sinusoidal sources.

Phasor

- **A complex number** z can be written in **rectangular form as**

$$z = x + jy$$

Where

- $j = \sqrt{-1}$
- **x** is the real part of z;
- **y** is the imaginary part of z.

Phasor

- The complex number z can also be written in polar or exponential form as

$$z = r\angle\phi = re^{j\phi}$$

Where

- $\boldsymbol{r}$ is the magnitude of z, and
- $\boldsymbol{\phi}$ is the phase of z.

Phasor

- ***Generally Z*** can be represented in three ways:

  a. Rectangular $z = x + jy = r(\cos\phi + j\sin\phi)$

  b. Polar $z = r\angle\phi$

  c. Exponential $z = re^{j\phi}$

Where $r = \sqrt{x^2 + y^2}$

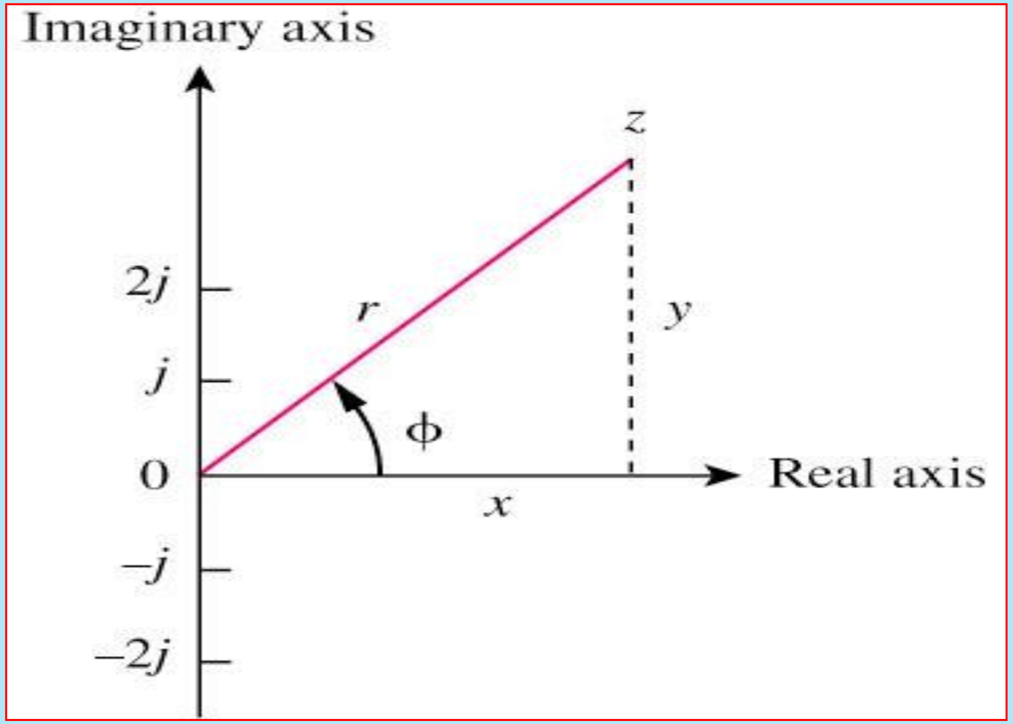
$\phi = \tan^{-1}\frac{y}{x}$

Z may be written as

$$z = x + jy = r\underline{/\phi} = r(\cos\phi + j\sin\phi)$$

Phasor

A complex number "z" graphed in the **complex plane** is shown below:



Representation of a complex number $z = x + jy$

Converting from rectangular to polar form

- We notice that "r" is the magnitude of the complex number "z".

$$r = \sqrt{x^2 + y^2}$$

- Also, by the definition of the tangent of an angle:

$$\phi = tan^{-1}\left(\frac{y}{x}\right)$$

Converting from polar to rectangular form

- Using the definition of sine and cosine of an angle gives us:

$$x = r\cos\phi$$

$$y = r\sin\phi$$

When working with complex numbers, it is useful to keep in mind the basic properties of mathematical operations performed on them:

Properties of Complex Numbers

Mathematic operation of complex number:

1. Addition $z_1 + z_2 = (x_1 + x_2) + j(y_1 + y_2)$
2. Subtraction $z_1 - z_2 = (x_1 - x_2) + j(y_1 - y_2)$
3. Multiplication $z_1 z_2 = r_1 r_2 \angle \phi_1 + \phi_2$
4. Division $\frac{z_1}{z_2} = \frac{r_1}{r_2} \angle \phi_1 - \phi_2$
5. Reciprocal $\frac{1}{z} = \frac{1}{r} \angle -\phi$
6. Square root $\sqrt{z} = \sqrt{r} \angle \phi/2$
7. Complex conjugate $z^* = x - jy = r\angle -\phi = re^{-j\phi}$
8. Euler's identity $e^{\pm j\phi} = \cos\phi \pm j\sin\phi$

Phasor

- **Transform a sinusoid to and from** the **time domain to the phasor domain:**

$$v(t) = V_m \cos(\omega t + \phi) \longleftrightarrow V = V_m \angle \phi$$

(Time domain) **(Phasor domain)**

Example Problems Involving Complex Numbers

Ex1) Evaluate the following **complex number** and **express the result in polar notation.**

$$(40\angle 50^{\circ} + 20\angle(-30^{\circ}))^{\frac{1}{2}}$$

Solution

Converting **to rectangular form:**

$$= \left[40cos50^{\circ} + j40sin50^{\circ} + 20cos(-30^{\circ}) + j20sin(-30^{\circ})\right]^{\frac{1}{2}}$$

Example Problems Involving Complex Numbers

❖ **Numerically evaluating the trig terms:**

$$= [43.03 + j20.64]^{\frac{1}{2}}$$

➢ **Recall that to convert to polar form:**

$$r = \sqrt{x^2 + y^2}$$

$$\phi = tan^{-1}\left(\frac{y}{x}\right)$$

Therefore we get:

$$= \left[\sqrt{43.03^2 + 20.64^2}\angle\left(tan^{-1}\left(\frac{20.64}{43.03}\right)\right)\right]^{\frac{1}{2}}$$

$$= [47.72\angle 25.63^{o}]^{\frac{1}{2}}$$

Example Problems Involving Complex Numbers

- Using the rule for complex numbers that involves square roots:

$$= \sqrt{47.72}\angle\left(\frac{25.63}{2}\right)$$

$$= 6.91\angle 12.82^{\circ}$$

Phasor Notation

- Phasor Notation/Representation is based on Euhler's Identity which states the following:

$$e^{\pm j\phi} = cos(\phi) \pm jsin(\phi)$$

- Notice that **cosine(phi)** and **sine(phi)** are the real and **imaginary** parts of the complex number:

$$cos(\phi) = R_e\{e^{j\phi}\} \qquad (eqn\ 1)$$

$$sin(\phi) = I_m\{e^{j\phi}\} \qquad (eqn\ 2)$$

Phasor Notation

- Either equation 1 or equation 2 can be used to develop a phasor, but
- Standard convention is to use equation #1 (the cosine term).

$$cos(\phi) = R_e\{e^{j\phi}\} \qquad (eqn\ 1)$$

$$sin(\phi) = I_m\{e^{j\phi}\} \qquad (eqn\ 2)$$

Developing the phasor notation

- **Given the following sinusoid:**

$$v(t) = V_m cos(\omega t + \phi)$$

- By equation #1 we can rewrite this as:

$$v(t) = R_e\{V_m e^{j(\omega t + \phi)}\}$$

and by rules of exponentials:

$$= R_e\{V_m e^{j\omega t} e^{j\phi}\} \qquad (Eqn3)$$

Developing the phasor notation

- Now let us define the following expression:

$$\mathbb{V} = V_m e^{j\phi} \qquad (expression\ 4)$$

- and recall the following expression for converting a complex number "z" from exponential form to polar form:

$$z = re^{j\phi} = r\angle\phi$$

- ...which means that expression #4 can be expanded in the following manner:

$$\mathbb{V} = V_m e^{j\phi} = V_m\angle\phi \qquad (expression\ 5)$$

Developing the phasor notation

- Using expression #5, we can now rewrite equation #3 as the following:

$$v(t) = R_e\{\mathbb{V}e^{j\omega t}\} \qquad (expression\ 6)$$

where the term "sinor" is defined as the following:

$$\mathbb{V}e^{j\omega t} = V_m e^{j(\omega t+\phi)}$$

Developing the phasor notation

- **To get the phasor that corresponds to a sinusoid:**

1. Express the sinusoid in cosine form so that it can be written as the real part of a complex number.

2. Remove the following time factor:

$$e^{j\omega t}$$

and whatever is left is the phasor.

Developing the phasor notation

- By suppressing the time factor, e(jwt), the sinusoid is transformed from the time domain to the phasor domain.

$$v(t) = V_m cos(\omega t + \phi), (time\ domain)$$

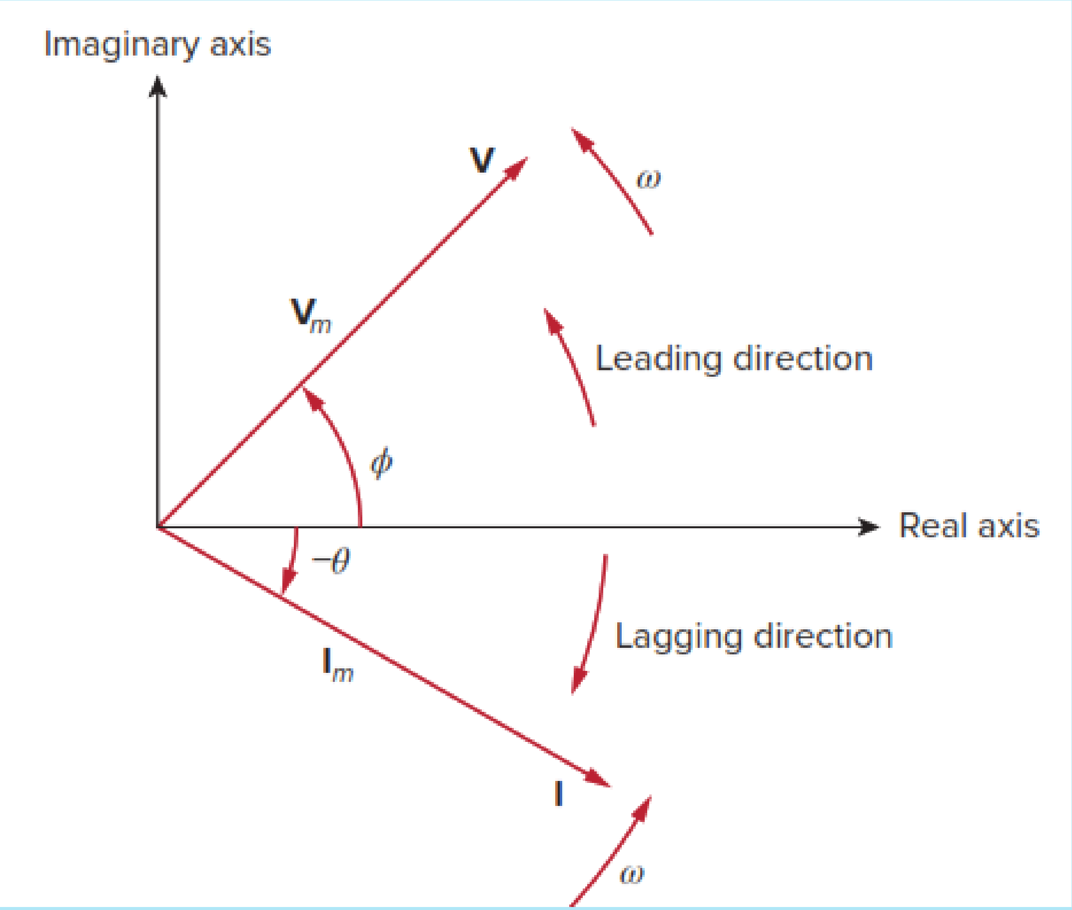
when transformed to the phasor domain is equivalent to

$$\mathbb{V} = V_m \angle \phi, (phasor\ domain)$$

Phasor

A phasor diagram showing

$$V = V_m \angle \phi$$
$$I = I_m \angle -\theta$$



Sinusoid-phasor transformation

Time domain representation	Phasor domain representation
$V_m\cos(\omega t + \phi)$	$V_m\underline{/\phi}$
$V_m\sin(\omega t + \phi)$	$V_m\underline{/\phi - 90^\circ}$
$I_m\cos(\omega t + \theta)$	$I_m\underline{/\theta}$
$I_m\sin(\omega t + \theta)$	$I_m\underline{/\theta - 90^\circ}$

The phasor domain is also known as the frequency domain.

Phasor

- **v(t)** is transformed to the phasor domain **jωV**

$$\frac{dv}{dt} \quad \Leftrightarrow \quad j\omega \mathbf{V}$$

(Time domain) (Phasor domain)

- The integral of v(t) is transformed to the phasor domain as V/jω

$$\int v\,dt \quad \Leftrightarrow \quad \frac{\mathbf{V}}{j\omega}$$

(Time domain) (Phasor domain)

Phasor

The differences between **v(t)** and **V** should be emphasized:

1. **V(t)** is time domain representation, while **V** is the frequency or phasor domain representation.

2. V(t) is time dependent, while V is not.

3. V(t) is always real with no complex term, while V is generally complex.

Example

1.Transform these sinusoids to phasors:

(a) $i = 6\cos(50t - 40°)$ A

(b) $v = -4\sin(30t + 50°)$ V

Solution:

(a) $i = 6\cos(50t - 40°)$ has the phasor

$$\mathbf{I} = 6\underline{/-40°}\ \text{A}$$

(b) Since $-\sin A = \cos(A + 90°)$,

$$v = -4\sin(30t + 50°) = 4\cos(30t + 50° + 90°)$$
$$= 4\cos(30t + 140°)\ \text{V}$$

The phasor form of v is

$$\mathbf{V} = 4\underline{/140°}\ \text{V}$$

Example

Evaluate these **complex numbers**:

(a) $(40\angle 50^\circ + 20\angle -30^\circ)^{1/2}$

For addition, convert to Rectangular

$$= (25.71 + j30.64 + 17.32 - j10)^{1/2}$$

$$= (43.03 + j20.64)^{1/2}$$

$$= (47.72\angle 25.63^\circ)^{1/2}$$

$$= (47.72)^{1/2} \frac{\angle 25.63^\circ}{2}$$

$$= 6.91\angle 12.81^\circ$$

$$\sqrt{z} = \sqrt{r}\angle \phi/2$$

Class work

(b) $\dfrac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*}$

Using polar-rectangular transformation, addition, multiplication, and division,

(b) $\dfrac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*}$

For addition, convert to Rectangular

$$\frac{10\angle -30^\circ + (3 - j4)}{(2 + j4)(3 - j5)^*} = \frac{8.66 - j5 + (3 - j4)}{(2 + j4)(3 - j5)^*} = \frac{11.66 - j9}{(2 + j4)(3 - j5)^*}$$

Taking care of Conjugate

$$= \frac{11.66 - j9}{(2 + j4)(3 + j5)}$$

For multiplication, convert to Polar

$$= \frac{11.66 - j9}{(4.47<63.43)(5.83<59)} = \frac{14.73\angle -37.66^\circ}{26.08\angle 122.47^\circ} = 0.565\angle -160.13^\circ$$

Home work

Evaluate the following complex number

(a) $[(5 + j2)(-1 + j4) - 5\angle 60^\circ]^*$

(b) $\dfrac{10 + j5 + 3\angle 40^\circ}{-3 + j4} + 10\angle 30^\circ + j5$

Homework solution

Solution:

(a) $[(5 + j2)(-1 + j4) - 5\angle 60°]^*$

= [(5.385<21.8)(4.123<104) - 5<60] *

= [22.2<125.8 - 5<60] *

= [-12.986 + j18 - (2.5 + j 4.33)] *

= [- 15.5 + j 13.67] *

= [- 15.5 − j 13.67]

(b) $\dfrac{10 + j5 + 3\angle 40°}{-3 + j4} + 10\angle 30° + j5$

= $\dfrac{10 + j5 + 2.298 + j1.93}{-3 + j4}$ + 8.66 + j5 + $j5$

= $\dfrac{12.298 + j6.928}{-3 + j4}$ + 8.66 + j10

= $\dfrac{14.12<29.39}{5< 126.86}$ + 8.66 + j10

= 2.83<-97.47 + 8.66 + j10

= 2.83<-97.47 + 8.66 + j10

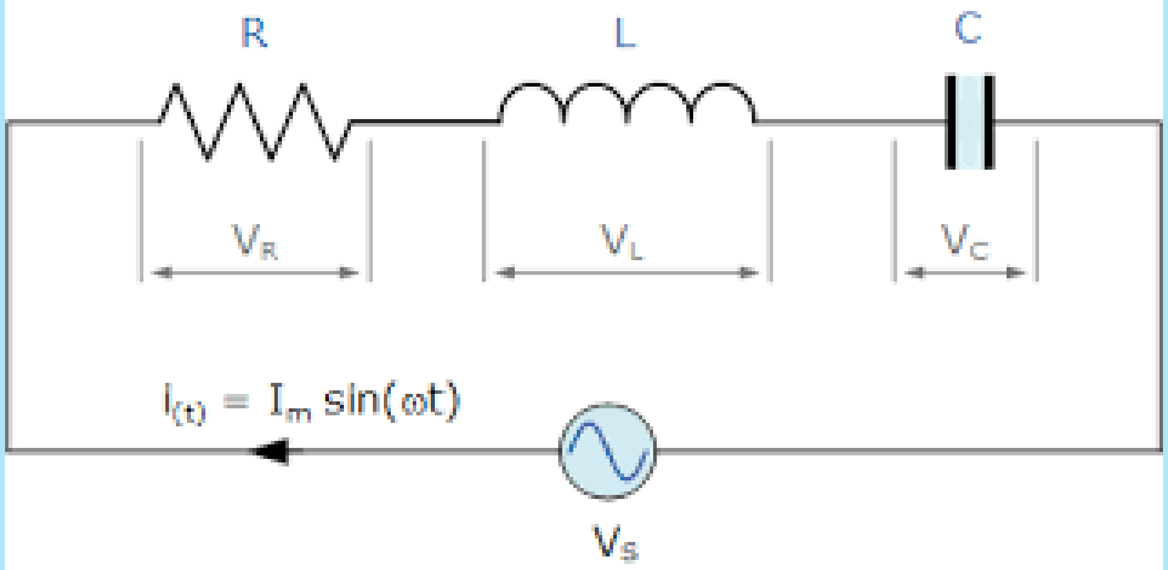
= - 0.367 - j 2.799 + 8.66 + j10

= 8.293 + j7.2

Impedance ,Admittance and Phasor diagrams

AC series circuit

- An AC circuit consists of three main components; a resistor, capacitor and an inductor which resist the flow of electric current in their own unique ways.



Impedance

- Determine how the resistance, capacitance, and inductance **"impede"** the current in ac circuit. Or
- It is the measure of overall opposition of an AC circuit to current denoted by Z
- The symbol for impedance is the letter Z.
- Unit is the ohm (Ω).
- The **polar form impedance** is written as: $Z = Z < \phi \ (\Omega)$
- The magnitude(in ohms) of the impedance is determined as: $Z = \sqrt{R^2 + X^2} \ (\Omega)$ $\phi = \pm \tan^{-1}(\frac{X}{R})$

Impedance

- The **rectangular form of impedance** is written as:

$$Z = R \pm jX,$$

Where

- R is **resistance** and X is **reactance** (X_L or X_C).

Note **Impedance can be split into two parts:**

Resistance R (a part which is constant regardless of frequency)

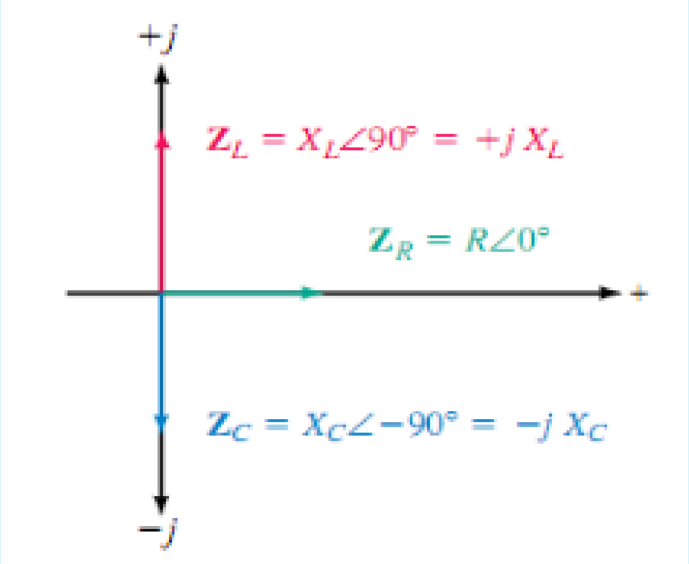
Reactance X (a part that varies with frequency due to capacitance and inductance)

Impedance

- **If we are given** the polar form of the impedance, then we may determine the **equivalent rectangular expression from as:**

$$R = Z\cos\phi \quad \text{and} \quad X = Z\sin\phi$$

Impedance diagram

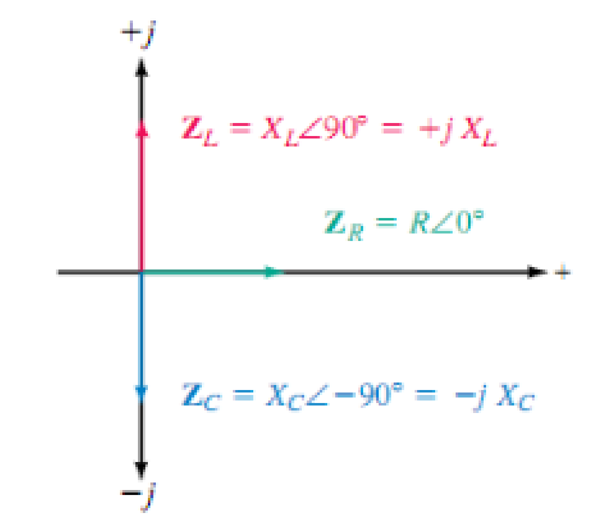


Impedance

Resistive impedance Z_R is a vector having a magnitude of R along **the positive real axis;**

Inductive impedance Z_L is a vector having a magnitude of X_L along the **positive imaginary.**

capacitive impedance Z_C is a vector having a magnitude of X_C along the **negative imaginary axis**



Mathematically, each of the vector impedance is written as follows

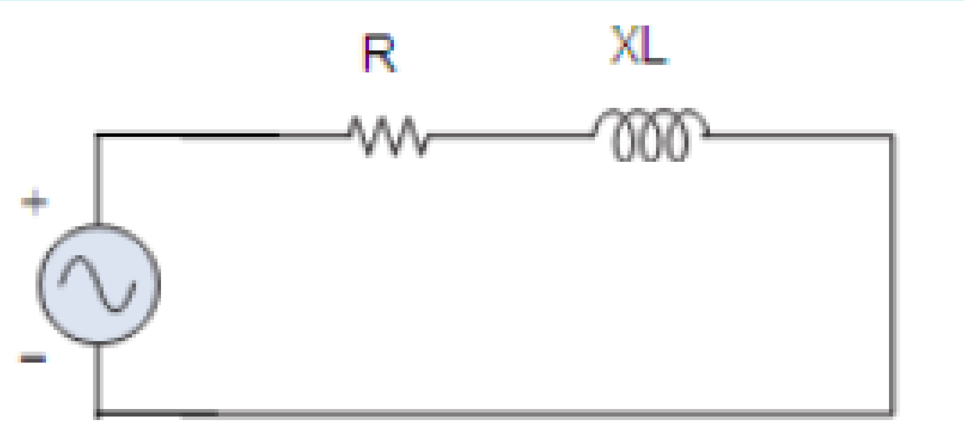
$$Z_R = R < 0^0 = R + j0 = R$$

$$Z_L = X_L < 90^0 = 0 + jX_L = jX_L$$

$$Z_C = X_C < -90^0 = 0 - jX_C = -jXc$$

R-L Circuit

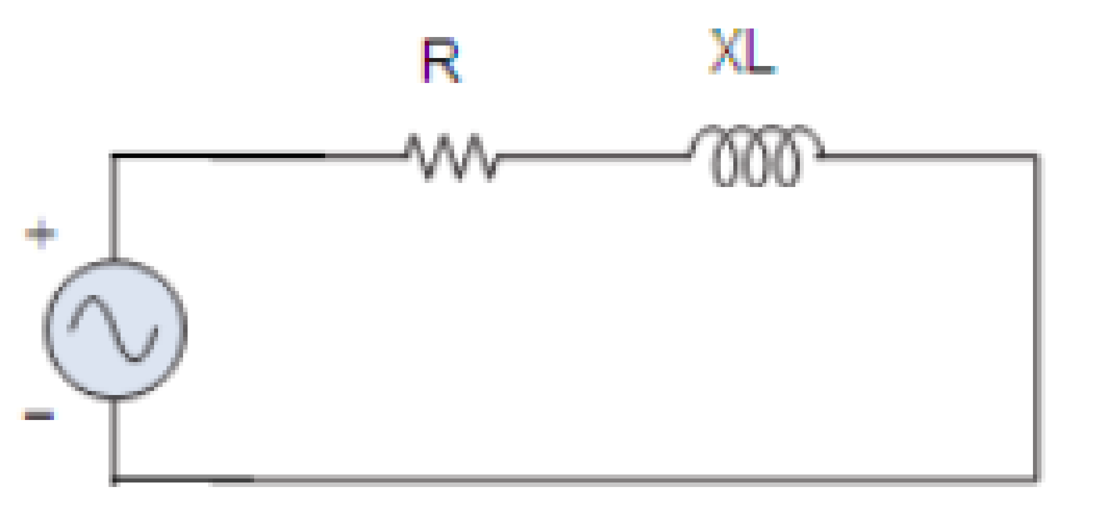
- **R-L circuit is** the **combination of resistive and inductive load.**



R- L circuit

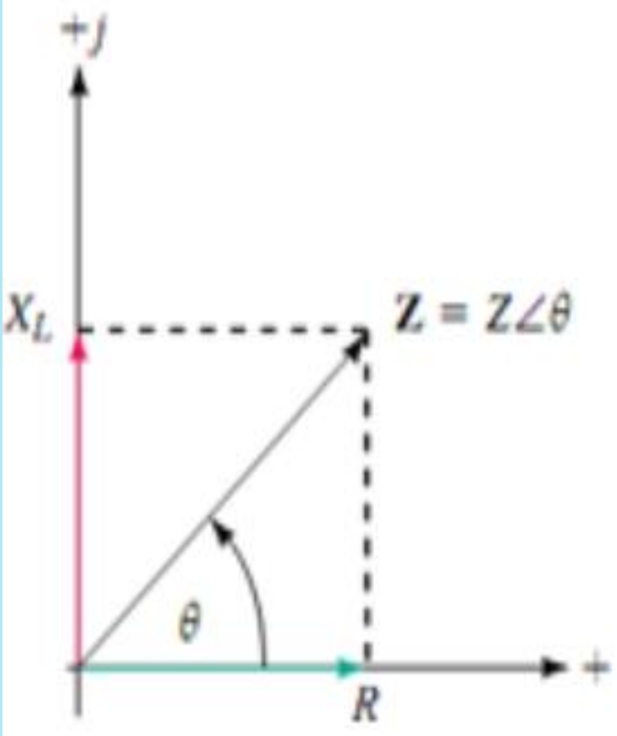
R-L Circuit

- In R-L circuit the **total impedance Z is**

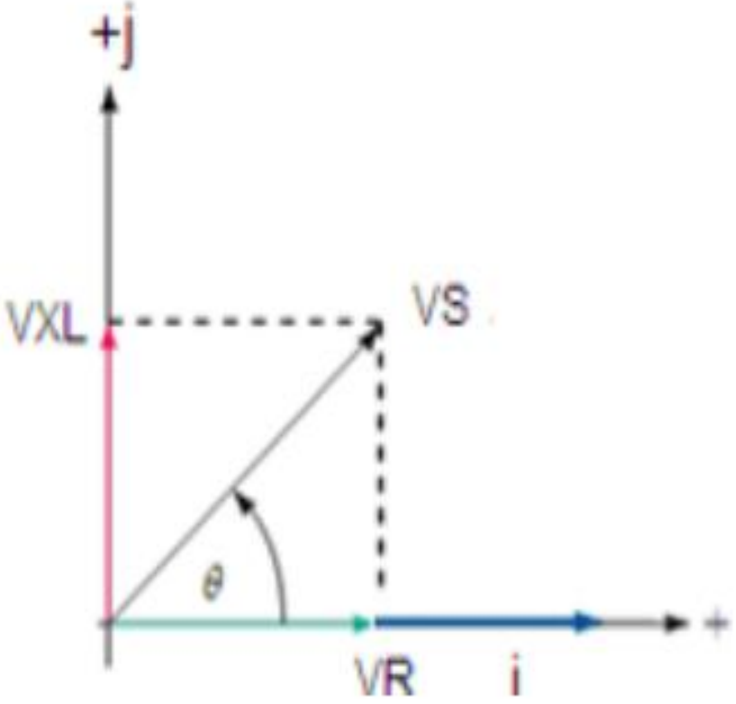


$$Z = R + jX_L or \quad Z = Z < \tan^{-1}(\frac{X_L}{R})$$

R-L Circuit



Impedance diagram



Phasor diagram

R-L Circuit

- **Voltage across resistor(R)** and **indictor (L) can be** determined as

$$V_R = i * Z_R$$

$$V_L = i * Z_L = i * X_L$$

- The **total voltage (supply voltage, Vs)**

$$V_s = V_R + jV_L$$

- **The total circuit current (i):**

$$i = \frac{V_s}{Z} = \frac{V_R + jV_L}{R + jX_L} \text{ or } \quad i = \frac{V_s < \phi_1}{Z < \phi_2} = \frac{V_s}{Z} < \phi_1 - \phi_2$$

Example

A **4Ω** resistor and a **9.55mH** inductor are connected in series with **240 V**, **50 Hz** AC source.

Calculate

a) **Inductive reactance**

b) **The impedance,**

c) **The total current, and**

d) **Draw impedance and phasor diagram.**

Example

- **Solution**

a. $inductive\ reactance, X_L = 2\pi f L = 2\pi(50)(9.55 * 10^{-3}) = 3\Omega$

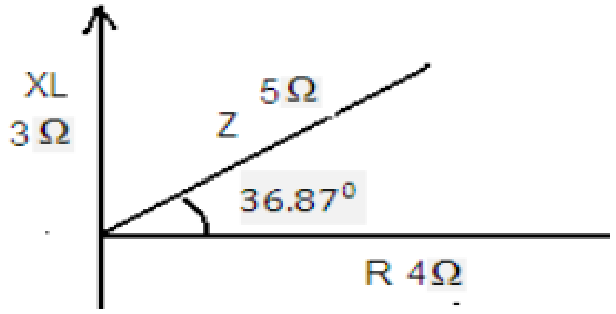
b. $impedance, Z = \sqrt{R^2 + {X_L}^2} = \sqrt{4^2 + 3^2} = 5\Omega$

c. $current, i = \frac{v}{z} = \frac{240V}{5\Omega} = 48A$

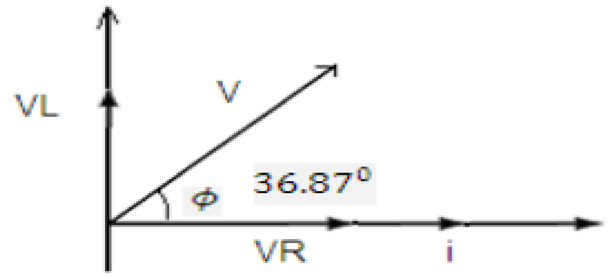
d. $\phi = \tan^{-1}\frac{X_L}{R} = \tan^{-1}\frac{3}{4} = 36.87^0\ lagging$

$$V_R = i * R = 48 * 4 = 192 < 0\ V$$

$$V_L = i * X_L = 48 * 3 = 144V\ but\ V_L = 144 < 90^0\ V$$



Impedance diagram



Phasor diagram

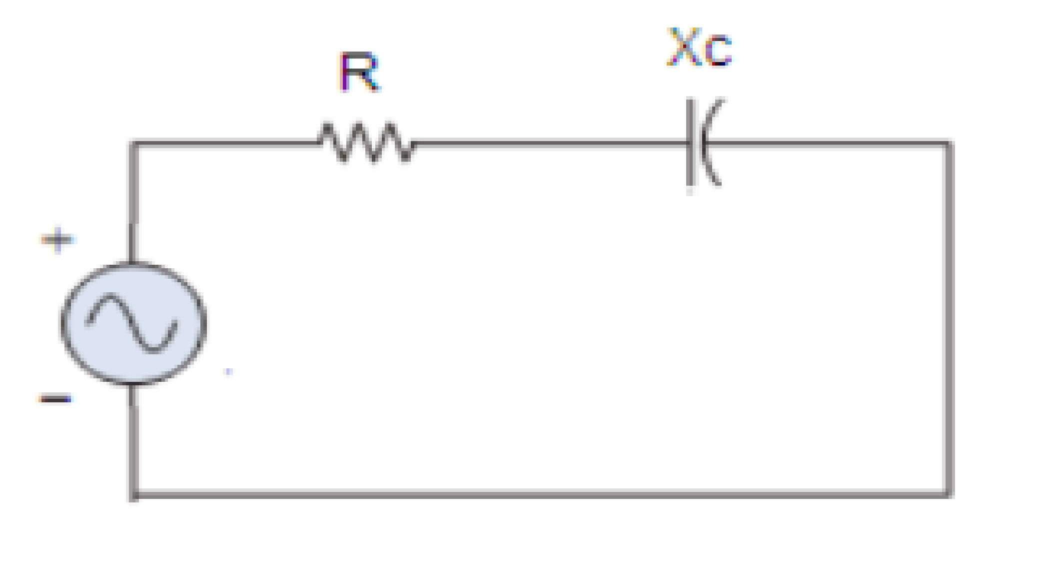
current lags voltage by 36.87

R-C Circuit

- An RC series circuit is an electrical circuit consisting of a resistor **R** and a capacitor **C** connected in series, driven by a voltage source or current source.

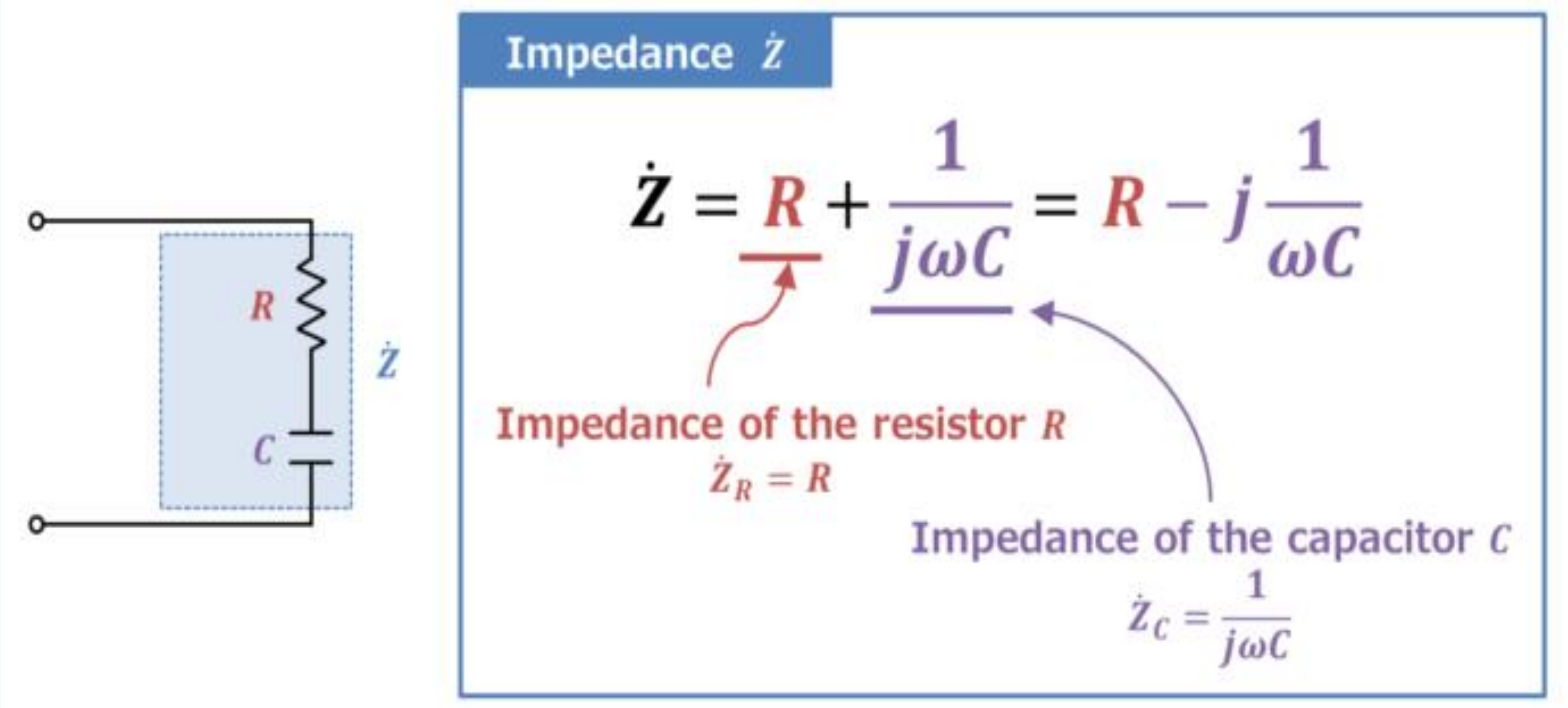
➢ In RC circuit the total impedance Z is written as:

$$Z = R - jX_C \quad or \quad Z = Z < \tan^{-1}(\frac{X_C}{R})$$



RC circuit

OR Impedance of the RC series circuit



Impedance of the RC series circuit

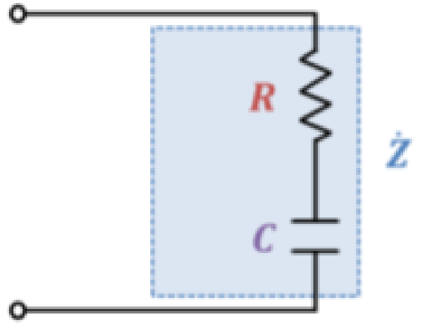
The impedance $Z^{\cdot}$ of the RC series circuit is the sum of the respective impedance, and is as follow:

$$\dot{Z} = \dot{Z}_R + \dot{Z}_C$$

$$= R + \frac{1}{j\omega C}$$

$$= R - j\frac{1}{\omega C}$$

Magnitude of the impedance of the RC series circuit



Magnitude Z of the impedance $\dot{Z}$

$$Z = |\dot{Z}| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$$

$$\dot{Z} = R - j\frac{1}{\omega C}$$

Adding the square of the real part R and the square of the imaginary part $\frac{1}{\omega C}$ and taking the square root

Vector diagram of the RC series circuit

The vector diagram of the impedance $\dot{Z}$ of the RC series circuit can be drawn in the following steps.

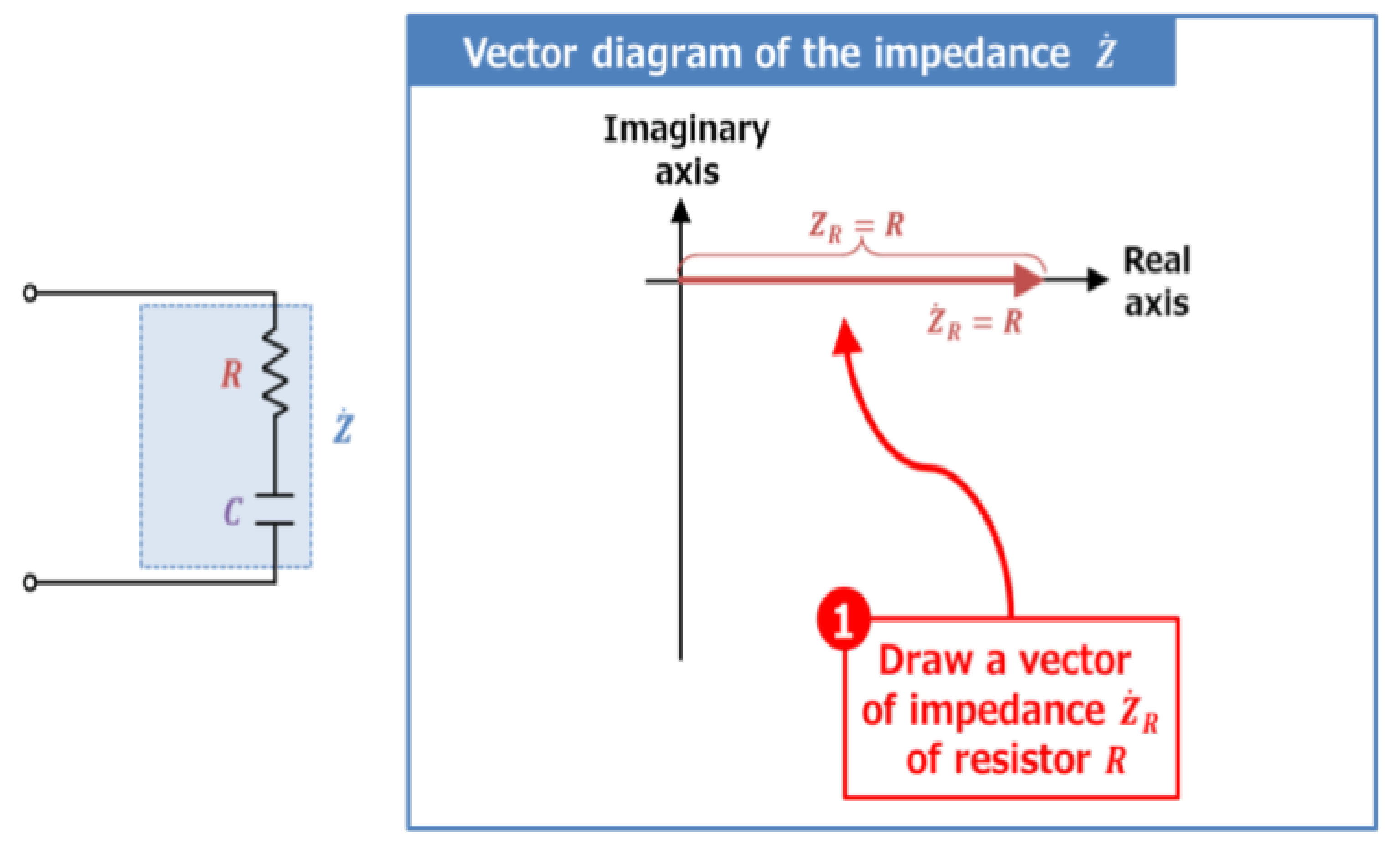
1. Draw a vector of impedance $\dot{Z}_R$ of resistor R
2. Draw a vector of impedance $\dot{Z}_C$ of capacitor C
3. Combine the vectors

Draw a vector of impedance $\dot{Z}_R$ of resistor R

The impedance $\dot{Z}_R$ of the resistor R is expressed as "$\dot{Z}_R=R$".

Therefore, the vector direction of the impedance $\dot{Z}_R$ is the direction of the real axis.

The magnitude (length) Z_R of the vector of the impedance $\dot{Z}_R$ is "$Z_R=|\dot{Z}_R|=R$".

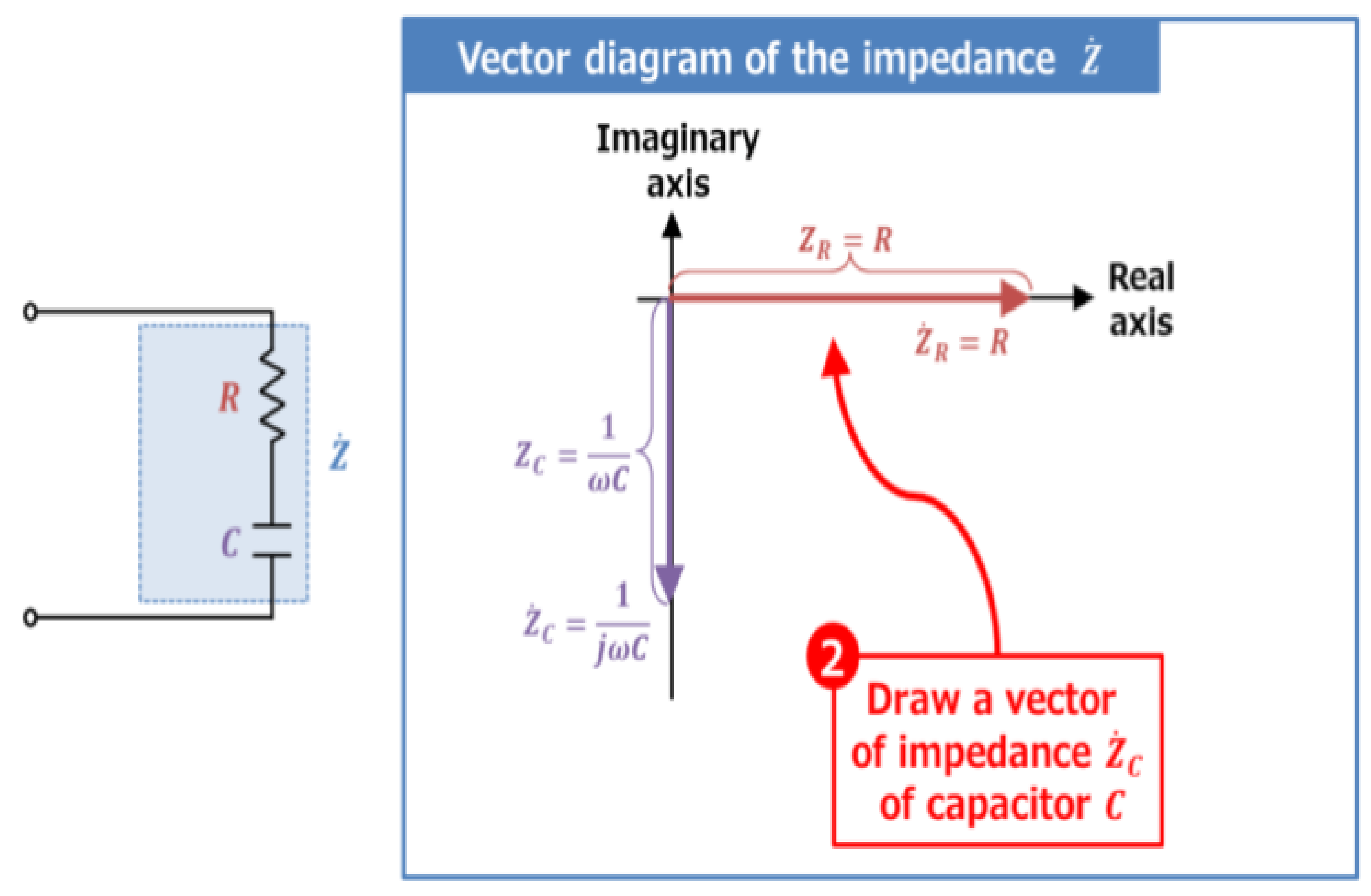


Draw a vector of impedance $\dot{Z}_C$ of capacitor C

The impedance $\dot{Z}_C$ of the capacitor C is expressed as **"$\dot{Z}_C = -j * 1/\omega C$".**

Therefore, the orientation of the impedance $\dot{Z}_C$ vector is **90° clockwise** around the real axis (with "−j", it rotates 90° clockwise).

The magnitude (length) Z_C of the vector of the impedance $\dot{Z}_C$ is "$Z_C = |\dot{Z}_C| = 1/\omega C$".

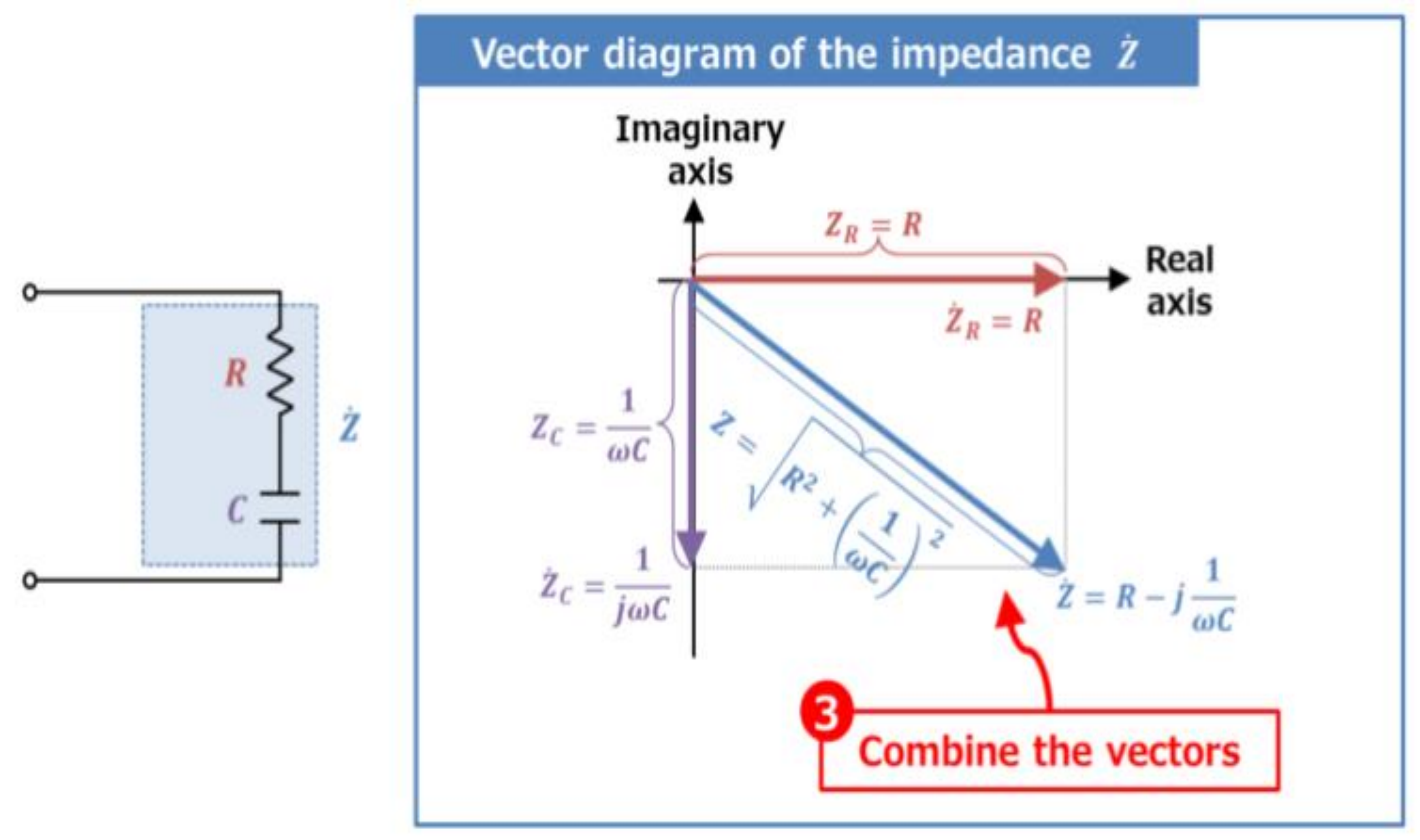


Combine the vectors

Combining the vector of "impedance $\dot{Z}_R$ of resistor R" and "impedance $\dot{Z}_C$ of capacitor C" is the vector diagram of the impedance $\dot{Z}$ of the RC series circuit.

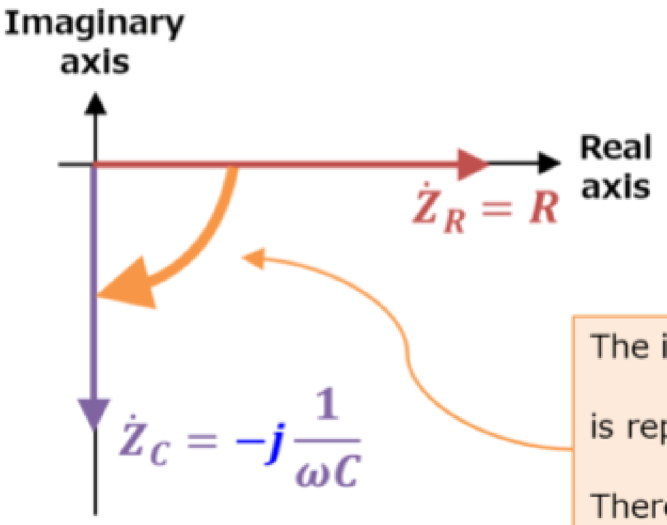
The magnitude (length) Z of the vector of the impedance $\dot{Z}$ is

$$"Z = |\dot{Z}| = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}".$$



Vector orientation

With "$+j$" is attached
☑The vector rotates 90° counterclockwise.
With "$-j$" is attached
☑The vector rotates 90° clockwise.



The impedance $\dot{Z}_C$ of capacitor C

is represented by "$\dot{Z}_C = -j\frac{1}{\omega C}$".

Therefore, the direction of vector $\dot{Z}_C$ is 90° clockwise around the real axis.

Vector orientation

When an imaginary unit "j" is added to the expression, the direction of the vector is rotated by 90°.

With "$+j$" is attached

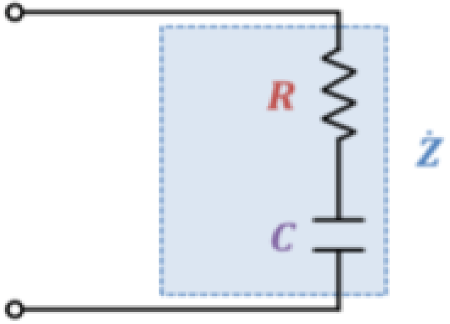
- The vector rotates **90° counterclockwise**.

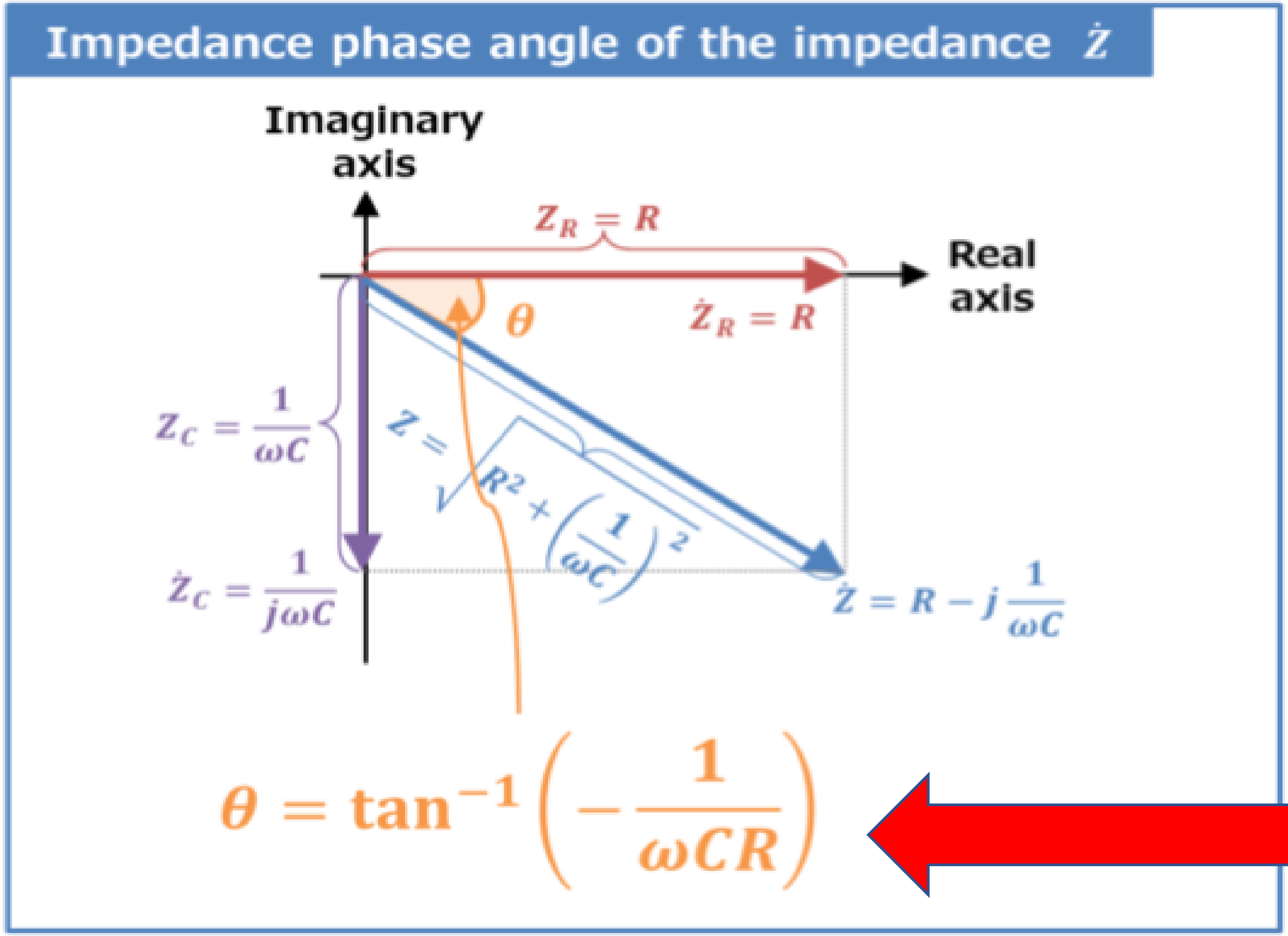
With "$-j$" is attached

- The vector rotates **90° clockwise**.

The impedance $\dot{Z}_C$ of capacitor C is represented by "$\dot{Z}_C = -j\frac{1}{\omega C}$". Therefore, the direction of vector $\dot{Z}_C$ is **90° clockwise** around the real axis.

Impedance phase angle of the RC series circuit





$$\tan\theta = \frac{-\frac{1}{\omega C}}{R}$$

$$\Leftrightarrow \theta = \tan^{-1}\left(-\frac{1}{\omega CR}\right)$$

Example

A resistor of **25Ω** is connected in series with a capacitor of **45μF**. **calculate**

(a) The impedance,

(b) The current taken from a 240,50Hz supply.

(c) Find also the phase angle between the supply voltage and the current.

Example

Solution

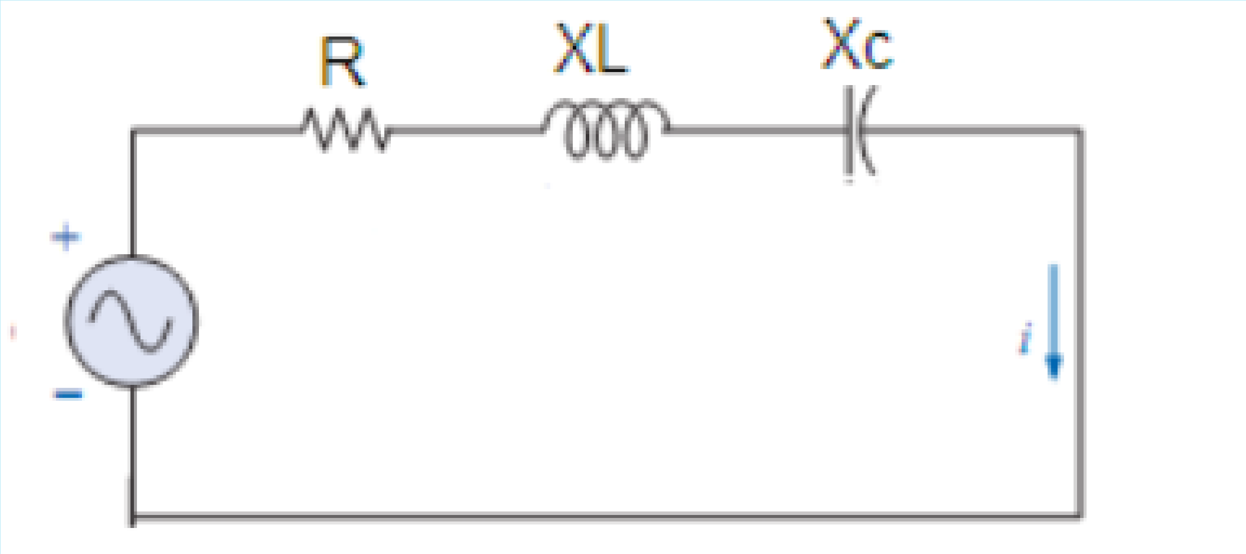
- Capacitive reactance, $X_c = \frac{1}{2\pi fC} = \frac{1}{2\pi(50)(45*10^{-6})} = 70.74\ \Omega$
- Impedance $Z = \sqrt{R^2 + X_C{}^2} = \sqrt{25^2 + 70.74^2} = 75.03\Omega$
- Current, $i = \frac{V}{Z} = \frac{240V}{75.03\Omega} = 3.2A$

Phase angle between the supply voltage and current

$$\phi = \tan^{-1}\frac{X_C}{R} = \frac{70.74}{25}$$

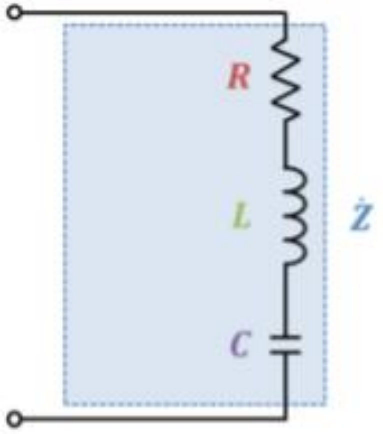
Series RLC circuit

- An RLC circuit is an electrical circuit consisting of a resistor (R), an inductor (L), and a capacitor (C), connected in series or in parallel.



RLC circuit

Impedance of the RLC series circuit



Impedance $\dot{Z}$

$$\dot{Z} = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Reactance of the inductor L

$X_L = \omega L$

Reactance of the capacitor C

$X_C = \dfrac{1}{\omega C}$

The impedance $\dot{Z}$ can be divided into the following three cases.

$X_L > X_C$	$X_L < X_C$	$X_L = X_C$
$\dot{Z} > 0$	$\dot{Z} < 0$	$\dot{Z} = 0$
$\dot{Z}$ is inductive.	$\dot{Z}$ is capacitive.	

Impedance of the RLC series circuit

The impedance $\dot{Z}_R$ of the resistor R, the impedance $\dot{Z}_L$ of the inductor L, and the impedance $\dot{Z}_C$ of the capacitor C can be expressed by the following equations:

$$\dot{Z}_R = R$$

$$\dot{Z}_L = jX_L = j\omega L$$

$$\dot{Z}_C = -jX_C = -j\frac{1}{\omega C} = \frac{1}{j\omega C}$$

Where

- ω is the angular frequency, which is equal to 2πf, and
- $X_L(=\omega L)$ is called **inductive reactance**, which is the resistive component of inductor L and
- $X_C(=1/\omega C)$ is called capacitive reactance, which is the resistive component of capacitor C.

Impedance of the RLC series circuit

The impedance $\dot{Z}$ of the RLC series circuit is the sum of the respective impedance, and is as follow:

$$\dot{Z} = \dot{Z}_R + \dot{Z}_L + \dot{Z}_C$$

$$= R + jX_L - jX_C$$

$$= R + j(X_L - X_C)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Impedance of the RLC series circuit

The impedance $Z^{\cdot}$ can be divided into the following three cases, **depending on the size of X_L and X_C.**

- **In Case $X_L > X_C$**
  - The impedance $\dot{Z}$ is positive($\dot{Z}>0$) and inductive.
- **In Case $X_L < X_C$**
  - The impedance $\dot{Z}$ is negative($\dot{Z}<0$) and capacitive.

Impedance of the RLC series circuit

In Case $X_L = X_C$

- The impedance $\dot{Z}$ is "$\dot{Z} = R$". In this case, the circuit is in series resonance. When series resonance is established, the angular frequency ω and frequency f are as follows:

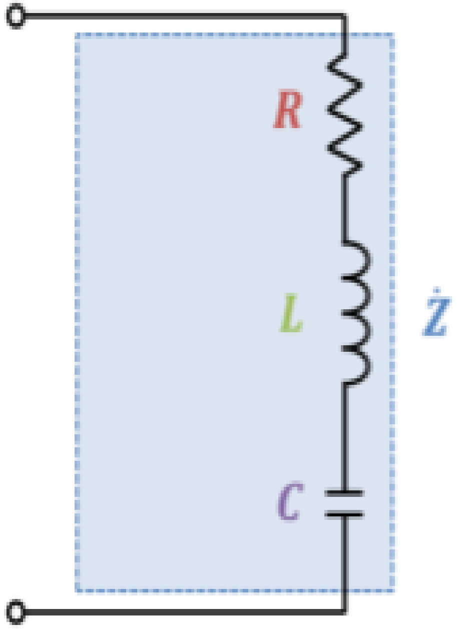
$$X_L = X_C$$

$$\omega L = \frac{1}{\omega C}$$

$$\Leftrightarrow \omega = \frac{1}{\sqrt{LC}}$$

$$\Leftrightarrow f = \frac{1}{2\pi\sqrt{LC}}$$

Magnitude of the impedance of the RLC series circuit



Magnitude Z of the impedance $\dot{Z}$

$$Z = |\dot{Z}| = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

The absolute value of

$$\dot{Z} = R + j(X_L - X_C) = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

Magnitude of the impedance of the RLC series circuit

The magnitude Z of the impedance $\dot{Z}$ of the RLC series circuit is the absolute value of "
$\dot{Z} = R + j\left(\omega L - \dfrac{1}{\omega C}\right)$".

In more detail, the magnitude Z of the impedance $\dot{Z}$ can be obtained by adding the square of the real part R and the square of the imaginary part $\omega L - \dfrac{1}{\omega C}$ and taking the square root, which can be expressed in the following equation.

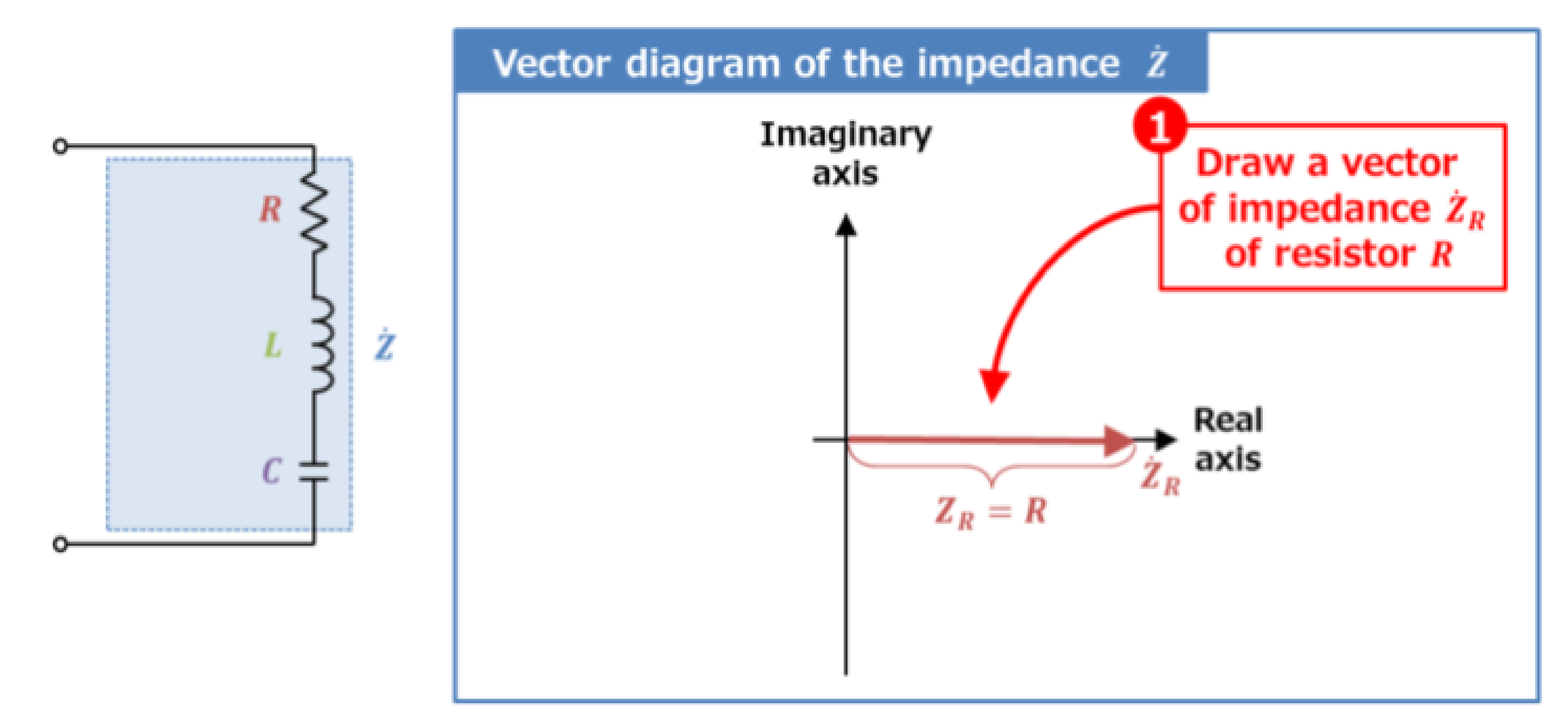
$$Z = |\dot{Z}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Vector diagram of the RLC series circuit

The vector diagram of the impedance $Z^{\cdot}$ of the RLC series circuit can be drawn in the following steps.

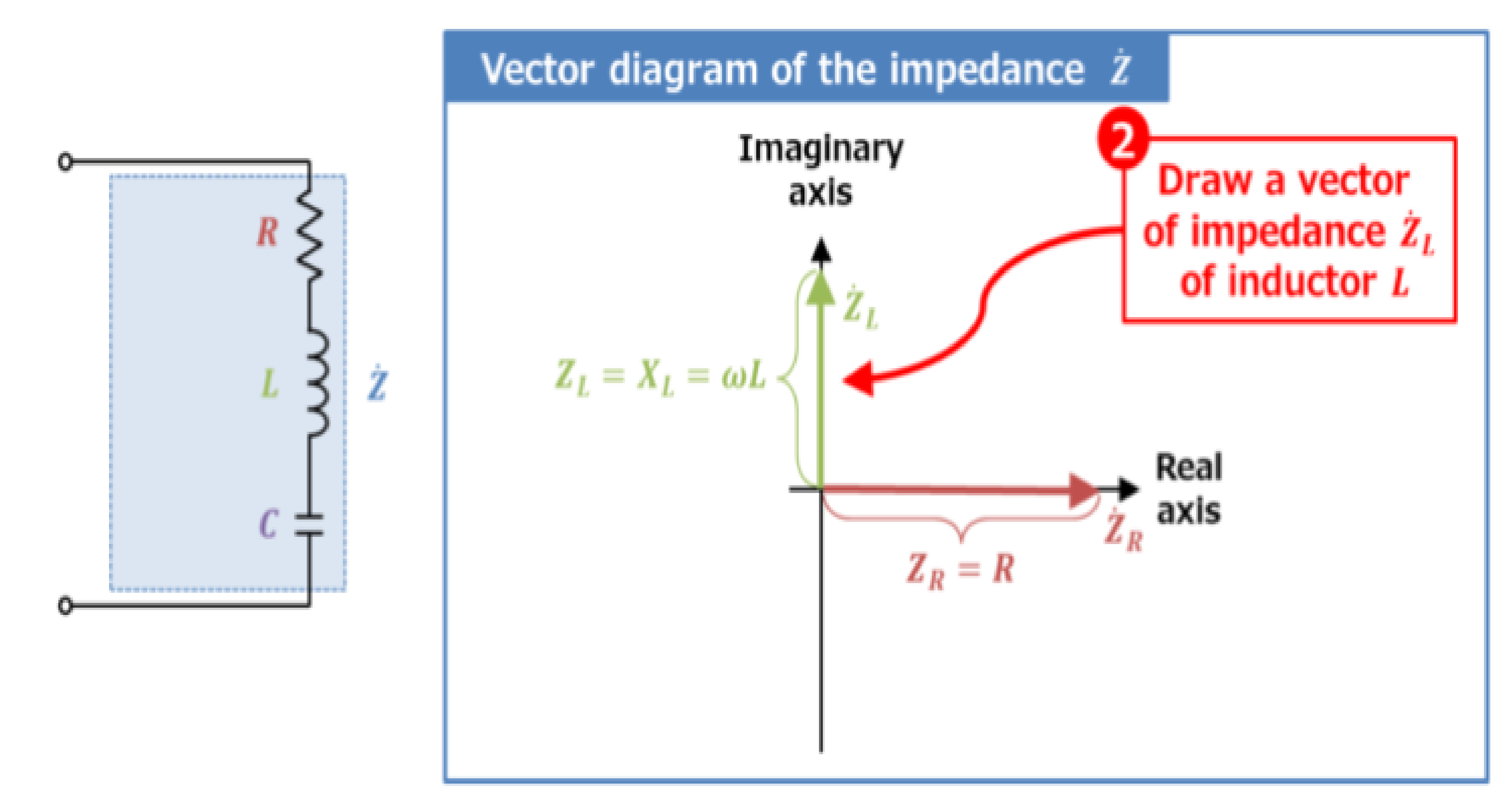
1. Draw a vector of impedance $\dot{Z}_R$ of resistor R
2. Draw a vector of impedance $\dot{Z}_L$ of inductor L
3. Draw a vector of impedance $\dot{Z}_C$ of capacitor C
4. Combine the vectors

Draw a vector of impedance Z_R of resistor R



The magnitude (length) Z_R of the vector of the impedance $\dot{Z}_R$ is "$Z_R = |\dot{Z}_R| = R$".

Draw a vector of impedance $\dot{Z}_L$ of inductor L



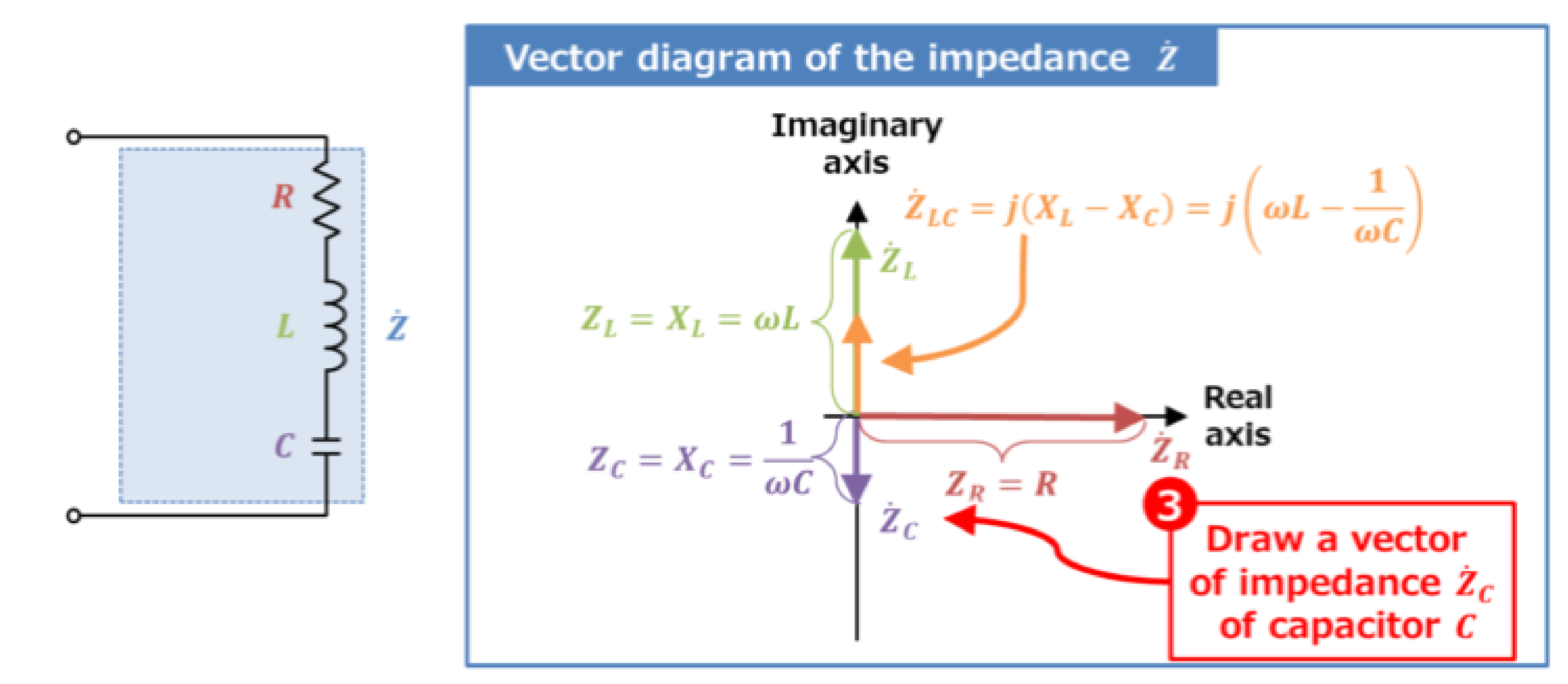
The impedance Z_L of the inductor L is expressed as "Z_L=jωL".

Draw a vector of impedance $\dot{Z}_L$ of inductor L

Therefore, the impedance Z_L vector is **90° counterclockwise** around the real axis (with "+j", it rotates 90° counterclockwise).

The magnitude (length) Z_L of the vector of the impedance $\dot{Z}_L$ is "$Z_L = |\dot{Z}_L| = \omega L$".

3. Draw a vector of impedance $\dot{Z}_C$ of capacitor C



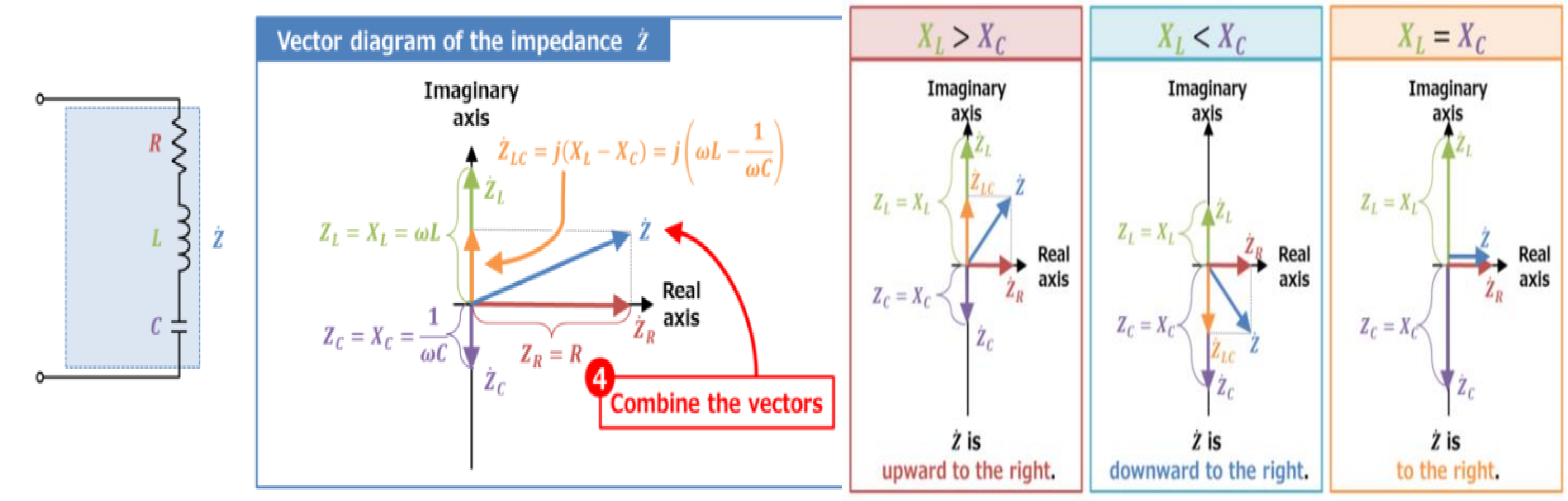
The impedance $\dot{Z}_C$ of the capacitor C is expressed as "$\dot{Z}_C = -j\frac{1}{\omega C}$".

Draw a vector of impedance $\dot{Z}_C$ of capacitor C

Therefore, the orientation of the impedance Z_C vector is **90° clockwise** around the real axis (with "−j", it rotates 90° clockwise).

The impedance $\dot{Z}_C$ of the capacitor C is expressed as "$\dot{Z}_C = -j\frac{1}{\omega C}$".

4. Combine the Vectors



Combine the Vectors

The **impedance $\dot{Z}$ of the RLC series circuit** is the sum of the respective impedance, and is as follow:

$$\dot{Z} = \dot{Z}_R + \dot{Z}_L + \dot{Z}_C$$

$$= R + jX_L - jX_C$$

$$= R + j(X_L - X_C)$$

$$= R + j\left(\omega L - \frac{1}{\omega C}\right)$$

The magnitude of X_L and X_C in the parentheses in the above equation changes the vector direction of the impedance $\dot{Z}$.

Combine the Vectors

In Case $X_L > X_C$

- The vector direction of the impedance $\dot{Z}$ is upward to the right.

In Case $X_L < X_C$

- The vector direction of the impedance $\dot{Z}$ is downward to the right.

In Case $X_L = X_C$

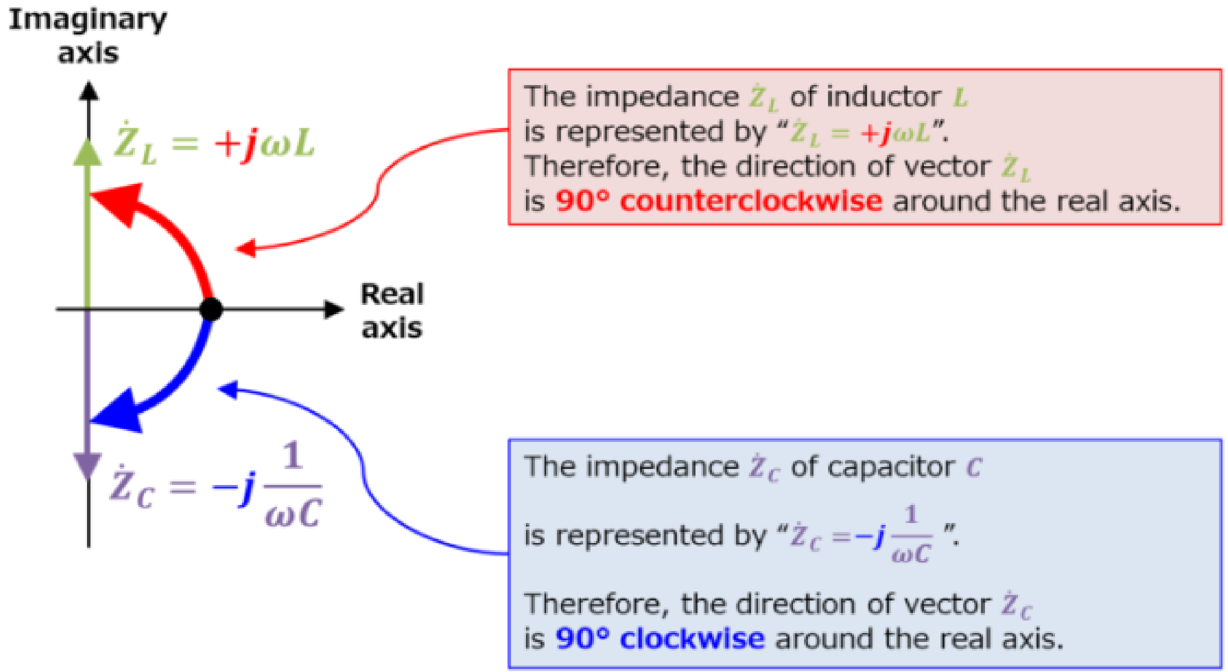
- Since the impedance $\dot{Z}$ is "$\dot{Z} = R$", the vector direction is to the right.

The magnitude (length) Z of the vector of the impedance $\dot{Z}$ can be expressed as follows.

$$Z = |\dot{Z}| = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}$$

Vector orientation

With "$+j$" is attached
☑The vector rotates 90° counterclockwise.
With "$-j$" is attached
☑The vector rotates 90° clockwise.



Vector orientation

When an imaginary unit "j" is added to the expression, the direction of the vector is rotated by 90°.

With "$+j$" is attached

- The vector rotates **90° counterclockwise**.

With "$-j$" is attached

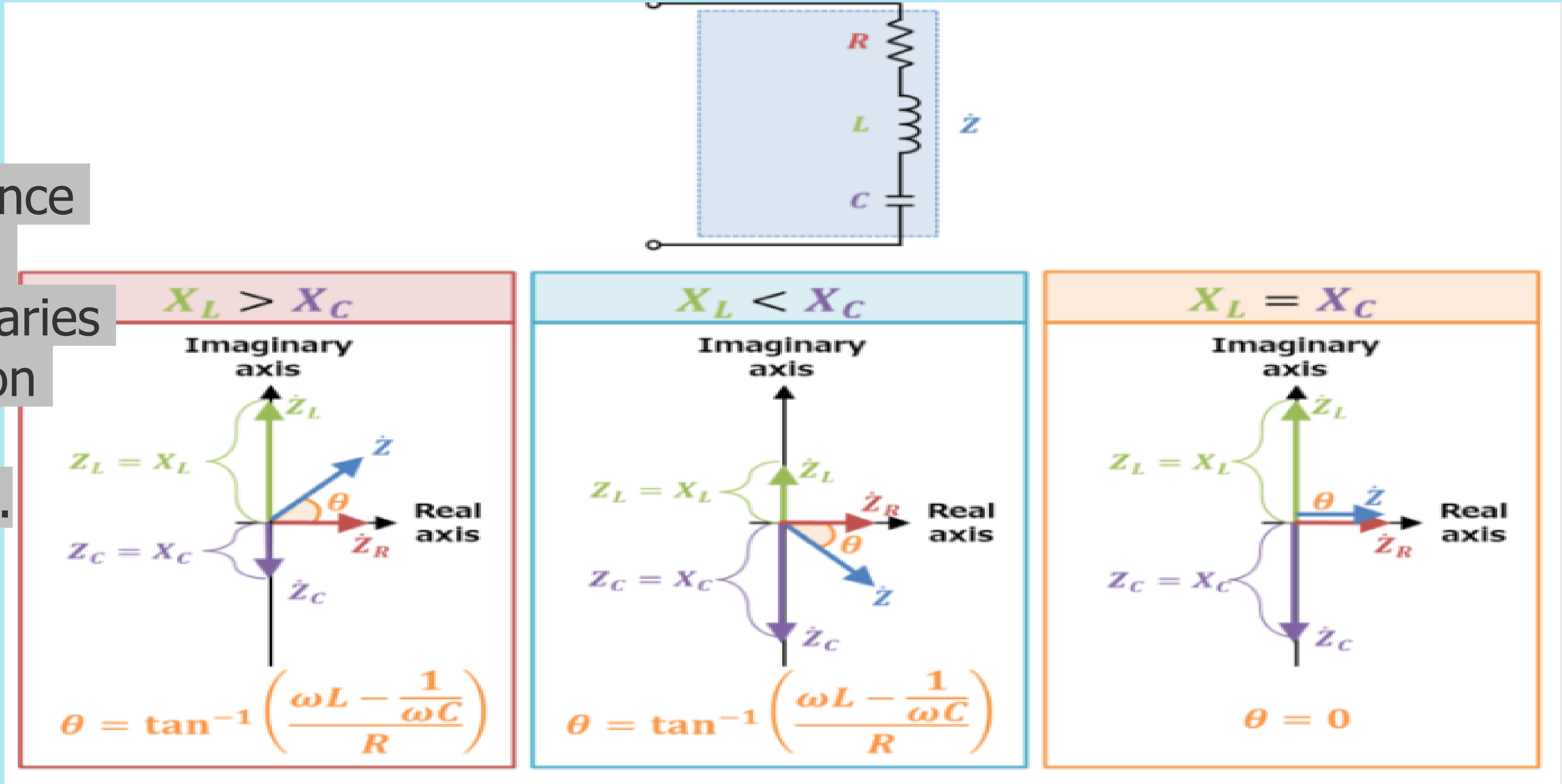
- The vector rotates **90° clockwise**.

The impedance $\dot{Z}_L$ of inductor L is represented by "$\dot{Z}_L = j\omega L$". Therefore, the direction of vector $\dot{Z}_L$ is **90° counterclockwise** around the real axis.

The impedance $\dot{Z}_C$ of capacitor C is represented by "$\dot{Z}_C = -j\frac{1}{\omega C}$". Therefore, the direction of vector $\dot{Z}_C$ is **90° clockwise** around the real axis.

Impedance phase angle of the RLC series circuit

The impedance phase angle ({\theta}) varies depending on the size of X_L and X_C.



Impedance phase angle of the RLC series circuit

In Case $X_L > X_C$

- The impedance phase angle θ is the following value:

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) [\mathrm{rad}]$$

The impedance angle ({\theta}) of the RLC series circuit is "**positive**".

In Case $X_L < X_C$

- The impedance phase angle θ is the following value:

$$\theta = \tan^{-1}\left(\frac{X_L - X_C}{R}\right) [\mathrm{rad}]$$

The impedance angle ({\theta}) of the RLC series circuit is "**negative**".

In Case $X_L = X_C$

- The impedance phase angle θ is the following value:

$$\theta = 0 [\mathrm{rad}]$$

Example

A 5Ω resistor, 120mH inductor and 100μF capacitor are connected in series to a 300V, 50Hz AC supply. Calculate

(a) the current flowing,

(b) the phase difference between the supply voltage and current,

(c) the voltage across the circuit elements, and

(d) draw the phasor and impedance diagram.

Example

Solution

$$X_L = 2\pi f L = 2\pi(50)(120 * 10^{-3}) = 37.70\Omega$$

$$X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi(50)(100 * 10^{-6})} = 31.83\Omega$$

Since X_L is greater than X_C the circuit is inductive

$$X_L - X_C = 37.7 - 31.83 = 5.87\Omega$$

$$impedance\ (Z) = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{5^2 + 5.87^2} = 7.71\Omega$$

a. $current\ (i) = \frac{v}{z} = \frac{300}{7.71} = 38.91A$

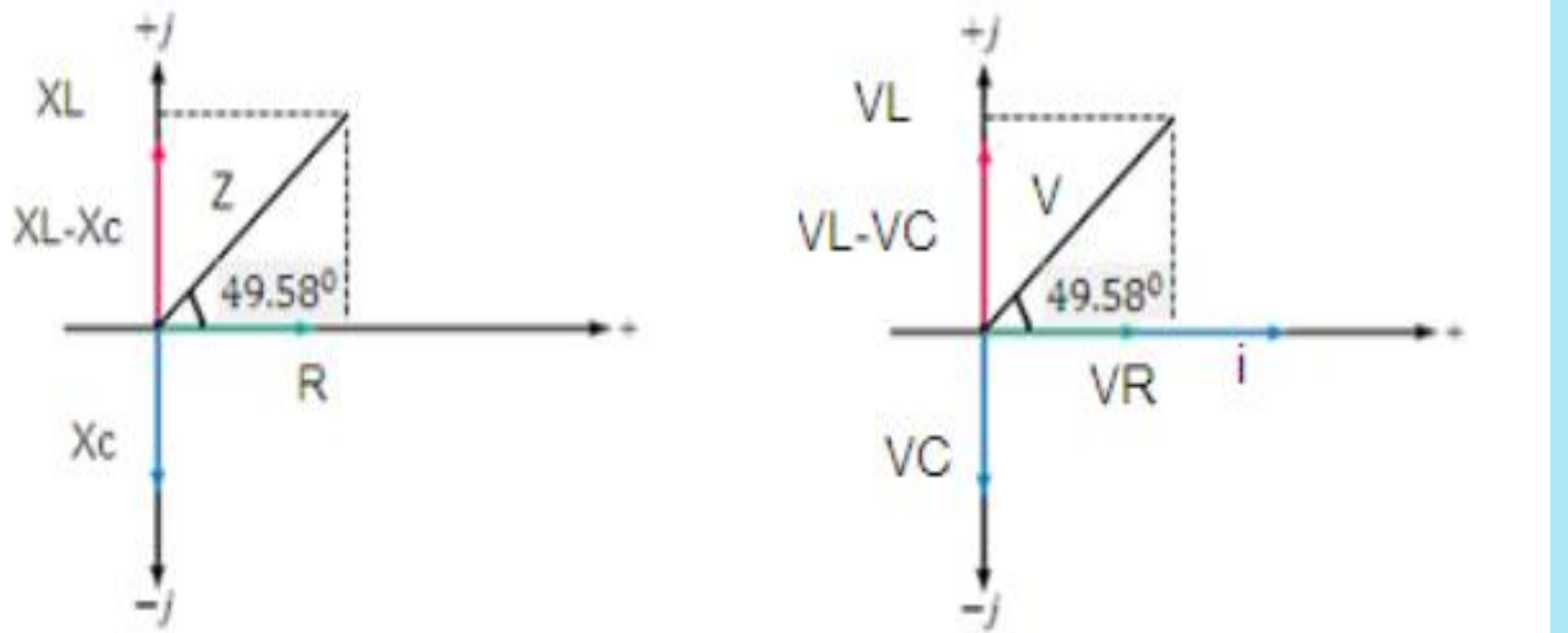
b. $phase\ angle\ \phi = \tan^{-1}(\frac{X_L - X_C}{R}) = \tan^{-1}(\frac{5.87}{5}) = 49.58^0$

c. $V_R = i * R = 38.91A * 5\Omega = 194.55V$

$V_L = i * X_L = 38.91 * 37.7 = 1466.9V$

$V_C = i * X_C = 38.91 * 31.83\Omega = 1238.5V$

Example



Impedance triangle

Phasor diagram

Home Work

- Parallel RLC circuit
  - ✓Impedance
  - ✓Admittance
  - ✓conductance
  - ✓Susceptance
  - ✓Phasor Diagram

https://electrical-information.com/rlc-series-circuit-impedance/

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Horaa Bulaa!

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ASTU

School of Electrical Engineering and Computing

Department of Electrical Power and Control Engineering

Fundamentals of Electrical Engineering (EPCE 2101)

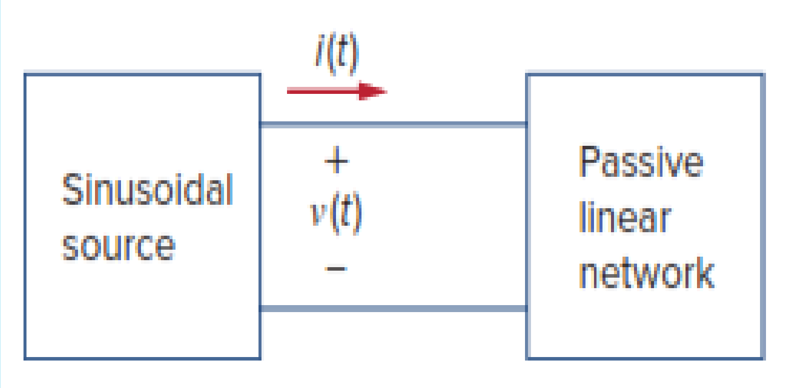
Chapter – 6
Steady State Power Analysis

Outlines

1. Instantaneous power
2. Average power
3. Effective or rms value
4. Apparent power and power factor
5. Complex power

1. Instantaneous power

- The **power p(t) absorbed by an element** is the product of voltage v(t) across the element and current i(t) through it. $P(t) = v(t).i(t)$
- **The instantaneous power** (in watts) is the power at any instant of time.



1. Instantaneous power

$$v(t) = V_m \cos(\omega t + \theta v)$$

$$i(t) = I_m \cos(\omega t + \theta i)$$

- **V_m and I_m** are the amplitudes (or peak values), and
- **θv and θi** are the phase angles of the voltage and current, respectively.

➢ The instantaneous power absorbed by the circuit is

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta v) \cos(\omega t + \theta i)$$

1. Instantaneous power

- We apply the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A - B) + \cos(A + B)]$$

$$p(t) = v(t)i(t) = V_m I_m \cos(\omega t + \theta v) \cos(\omega t + \theta i)$$

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

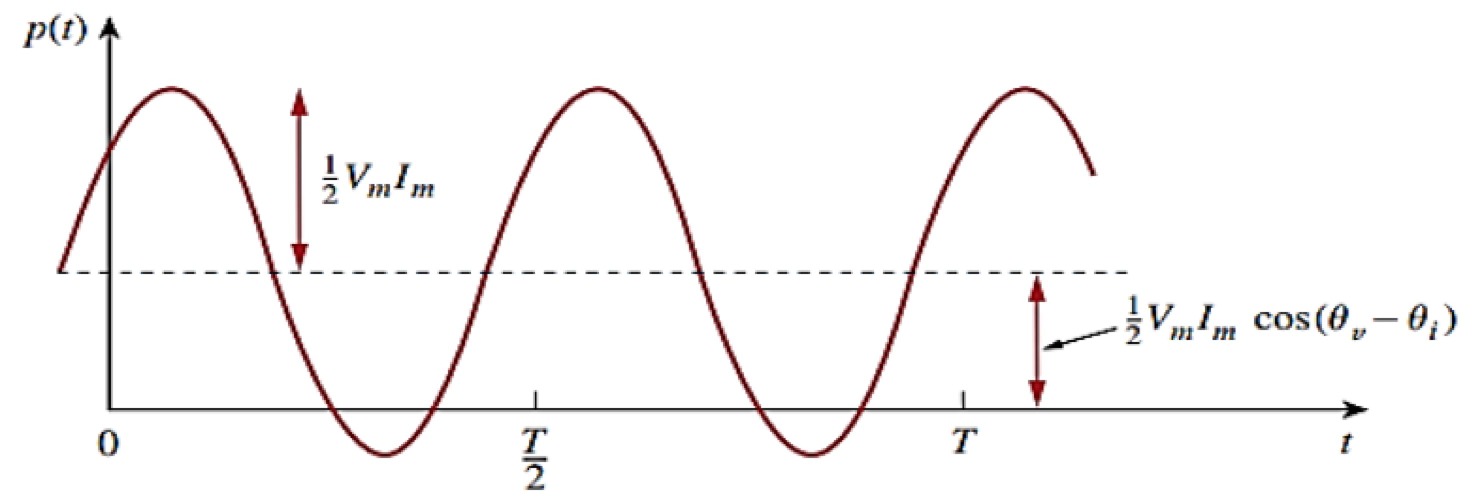
1. Instantaneous power

$$p(t) = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

- The **instantaneous power has two parts**.

1. The first part is **constant or time independent**. Its value depends on the phase difference between the voltage and the current.
2. The second part is **a sinusoidal** function whose frequency is which is **twice the angular frequency** of the voltage or current.

1. Instantaneous power



We observe that p(t) is positive for some part of each cycle and negative for the rest of the cycle.

- ❖ When p(t) is positive, power is absorbed by the circuit.
- ❖ When p(t) is negative, power is absorbed by the source

2.Average power

- Is the average of the instantaneous power over one period. Thus, the average power is given by

$$P = \frac{1}{T}\int_0^T p(t)\,dt$$

$$P = \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\,dt + \frac{1}{T}\int_0^T \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)\,dt$$

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)\frac{1}{T}\int_0^T dt + \frac{1}{2}V_m I_m \frac{1}{T}\int_0^T \cos(2\omega t + \theta_v + \theta_i)\,dt$$

2.Average power

- The **first integrand is constant**, and the average of a constant is the same constant.
- **The second integrand is a sinusoid**. We know that the average of a sinusoid over its period is zero because the area under the sinusoid during a **positive half-cycle** is canceled by the area under it during the following **negative half-cycle**.

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

2.Average power

- **To use phasors**, we notice that

$$\frac{1}{2}\mathbf{VI}^* = \frac{1}{2}V_m I_m \underline{/\theta_v - \theta_i} = \frac{1}{2}V_m I_m[\cos(\theta_v - \theta_i) + j\sin(\theta_v - \theta_i)]$$

- The average power P thus becomes

$$P = \frac{1}{2}\text{Re}[\mathbf{VI}^*] = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$

- **When θv = θi, the voltage and current are in phase**. This implies a purely resistive circuit or resistive load R, and

$$P = \frac{1}{2}V_m I_m = \frac{1}{2}I_m^2 R = \frac{1}{2}|\mathbf{I}|^2 R$$

2.Average power

- When **θv - θi = ±90**°, we have a purely reactive circuit, and

$$P = \frac{1}{2} V_m I_m \cos 90^\circ = 0$$

summary,

➢ A resistive load (R) absorbs power at all times, while a reactive load (L or C) absorbs zero average power.

Example

Given that

$v(t) = 120\cos(377t + 45°)$ V and $i(t) = 10\cos(377t - 10°)$ A

Find the **instantaneous power** and the **average power** absorbed by the passive linear network.

Solution

- The **instantaneous power** is given by

$$p = vi = \mathbf{1200} \cos(377t + 45°) \cos(377t - 10°)$$

Applying the trigonometric identity

$$\cos A \cos B = \frac{1}{2}[\cos(A + B) + \cos(A - B)]$$

$$\mathbf{p = 600[\cos(754t + 35°) + \cos 55°]}$$

$$\mathbf{p(t) = 344.2 + 600 \cos(754t + 35°)\ W}$$

The average power is

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2} 120(10) \cos[45° - (-10°)]$$

$$= 600 \cos 55° = 344.2\ \text{W}$$

which is the **constant part of p(t) above**.

Homework

1.Calculate the average power absorbed by an impedance Z = 30 − j70 Ω voltage $V = 120\angle 0^0$ is applied across it.

Solution: The current through the impedance is

$$\mathbf{I} = \frac{\mathbf{V}}{\mathbf{Z}} = \frac{120\underline{/0^\circ}}{30 - j70} = \frac{120\underline{/0^\circ}}{76.16\underline{/-66.8^\circ}} = 1.576\underline{/66.8^\circ}\text{ A}$$

The average power is

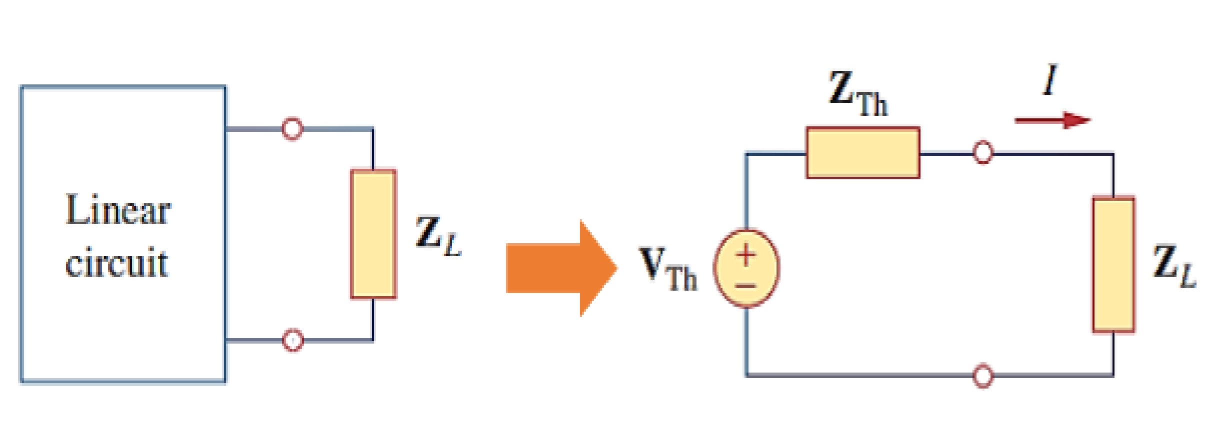
$$P = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) = \frac{1}{2}(120)(1.576)\cos(0 - 66.8^\circ) = 37.24\text{ W}$$

Maximum Average Power Transfer

- In DC circuit analysis we solved the problem of maximizing the power delivered by a supplying resistive network to a load.
- This is done by representing the circuit by its **Thevenin power-equivalent,** ***maximum power would be delivered to the load if the load resistance is equal to the Thevenin resistance***.

Maximum Average Power Transfer

- We now extend that result to ac circuits.



$$\mathbf{Z}_{\text{Th}} = R_{\text{Th}} + jX_{\text{Th}}$$

$$\mathbf{Z}_L = R_L + jX_L$$

$$\mathbf{I} = \frac{\mathbf{V}_{\text{Th}}}{\mathbf{Z}_{\text{Th}} + \mathbf{Z}_L} = \frac{\mathbf{V}_{\text{Th}}}{(R_{\text{Th}} + jX_{\text{Th}}) + (R_L + jX_L)}$$

Maximum Average Power Transfer

For maximum average power transfer, the load impedance $\mathbf{Z_L}$ must be equal to the complex conjugate of the Thevenin impedance $\mathbf{Z_{Th}}$.

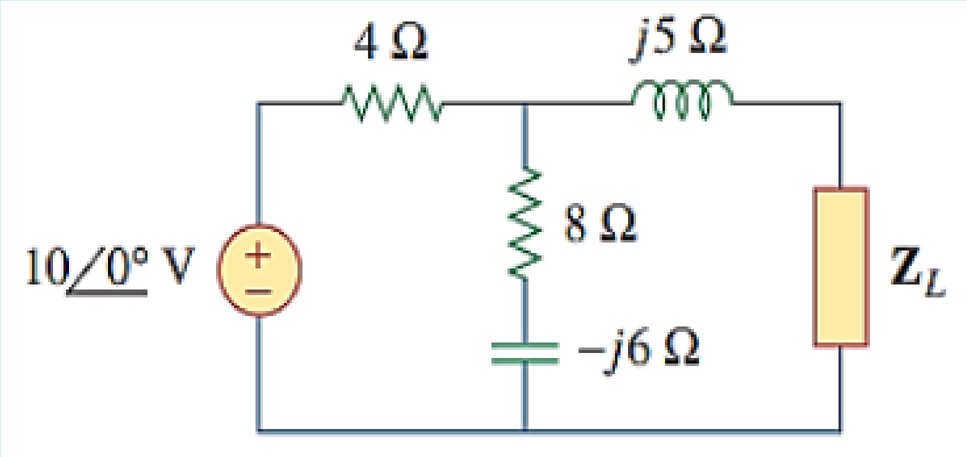
$$\mathbf{Z}_L = R_L + jX_L = R_{\text{Th}} - jX_{\text{Th}} = \mathbf{Z}_{\text{Th}}^*$$

The maximum transfer power is given by

$$P_{\max} = \frac{|\mathbf{V}_{\text{Th}}|^2}{8R_{\text{Th}}}$$

Homework

Determine the load impedance that maximizes the average power drawn from the circuit shown below. What is the maximum average power?



solution

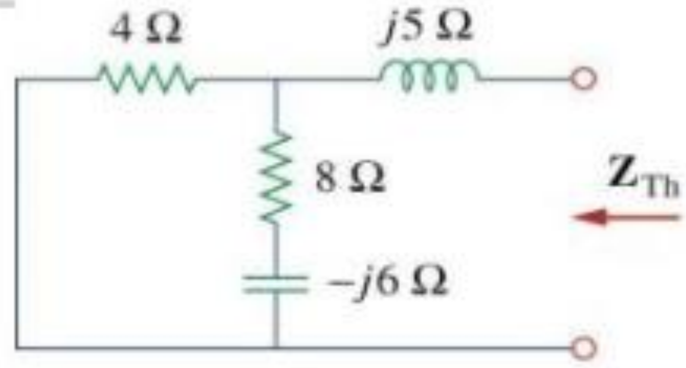
First we obtain the Thevenin equivalent

To find Z_{th}, consider circuit (a)

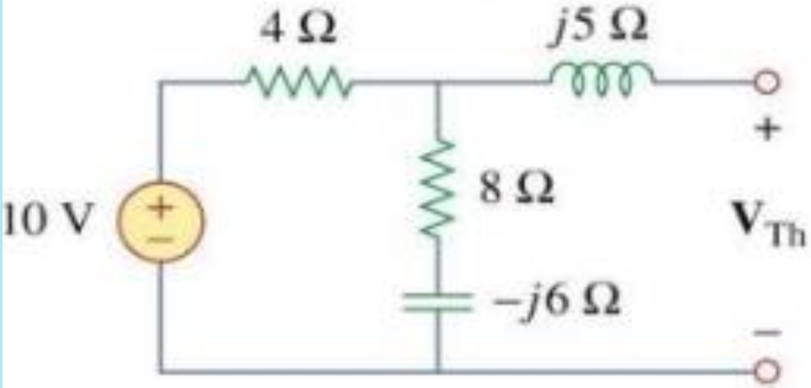
$$Z_{Th} = j5 + 4 \| (8 - j6)$$
$$= (2.933 + j4.467)\ \Omega$$

To find V_{th}, consider circuit (b)

$$V_{Th} = \frac{(8 - j6)}{4 + (8 - j6)}(10\angle 0^{\circ})$$
$$= 7.454\angle -10.3^{\circ}\ \text{V}$$



(a)



(b)

solution

From the result obtained, the load impedance draws the maximum power from the circuit when

$$Z_L = Z_{Th}^* = (2.933 - j4.467)\Omega$$

The maximum average power is

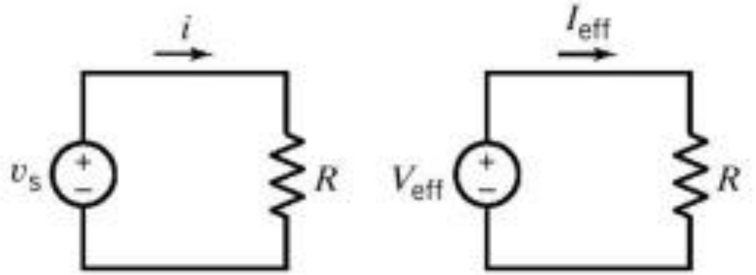
$$P_{max} = \frac{|V_{Th}|^2}{8R_{Th}} = \frac{(7.454)^2}{8(2.933)} = 2.368\text{W}$$

3. Effective or rms value

- The idea of effective value arises from the need to measure the **effectiveness of a voltage or current source in delivering power to a resistive load**.
- *The effective value of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.*
- **The average power absorbed** by the resistor in the ac circuit is

$$P = \frac{1}{T}\int_0^T i^2 R\, dt = \frac{R}{T}\int_0^T i^2\, dt$$

3. Effective or rms value



The goal is to find a dc voltage, V_{eff} (or dc current, I_{eff}), for a specified $v_s(t)$ that will deliver *the same average power* to *R* as would be delivered by the ac source.

The energy delivered in a period T is

$$W = PT$$

The average power delivered to the resistor by a periodic current is

$$P = \frac{1}{T}\int_0^T i^2 R\,dt$$

The power delivered by a direct current is

$$P = I_{eff}^2 R$$

$$\therefore P = \frac{1}{T}\int_0^T i^2 R dt = I_{eff}^2 R$$

Solve for I_{eff}

$$I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2 dt}$$

$$= I_{rms}$$ *rms = root-mean-square*

The ***effective value*** of a current is the steady current (dc) that transfer *the same average power* as the given time varying current.

3. Effective or rms value

- $I_{rms} = Im / \sqrt{2}$
- $V_{rms} = Vm / \sqrt{2}$

- The **average power** can be written in terms of the **rms values**.

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} \cos(\theta_v - \theta_i)$$

$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i)$$

Apparent power(S) and power factor

- If the voltage and current at the terminals of a circuit are

$$v(t) = V_m \cos(\omega t + \theta v) \text{ and } i(t) = I_m \cos(\omega t + \theta i),$$

Average power is

$$P = 1/2\,(V_m I_m \cos(\theta v - \theta i)),$$

$$P = V_{rms}\, I_{rms} \cos(\theta v - \theta i) = S \cos(\theta v - \theta i)$$

The average power is a product of two terms.

- The product Vrms and Irms is known as the **apparent power S**.
- The factor **cos (θv - θi)** is called the power factor (pf).

Apparent power and power factor

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) = S \cos(\theta_v - \theta_i)$$

- Power factor is the cosine of the phase difference between voltage and current .

The power factor is dimensionless, since it is the ratio of the average power to the apparent,

$$\text{pf} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

The angle **θv - θi is called the power factor angle**, since it is the angle, whose cosine is the power factor.

Apparent power and power factor
Note

- The **range of pf** is between **zero and unity**.
- For a purely resistive load, the voltage and current are in phase so that
  - **$\theta v - \theta i = 0$ and pf =1, the apparent power is equal to average power.**
- For a purely reactive load ,
  - $\theta v - \theta i = \pm 90^{\circ}$ and pf=0, the average power is zero.
- Leading power factor means that current leads voltage,
  - which implies **a capacitive load**. **[ICE]**
- Lagging power factor means that **current lags voltage**,
  - implying an **inductive load. [ELI]**

Group Activity

A series-connected load draws a current **$i(t)=4\cos(100\pi t + 10^\circ)A$** when the applied voltage is **$v(t)= 20\cos(100\pi t - 20^\circ)V$**. Find the **apparent power** and the **power factor** of the load. Determine the element values that form the series-connected load.

Solution

- The apparent power is → $S = V_{rms} I_{rms} = \frac{120}{\sqrt{2}} \frac{4}{\sqrt{2}} = 240 \text{ VA}$

The power factor is → $\text{pf} = \cos(\theta_v - \theta_i) = \cos(-20^\circ - 10^\circ) = 0.866 \qquad \text{(leading)}$

- The pf is leading because the current leads the voltage. The pf may also

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{120\angle -20^\circ}{4\angle 10^\circ} = 30\angle -30^\circ = 25.98 - j15\ \Omega$$

$$\text{pf} = \cos(-30^\circ) = 0.866 \qquad \text{(leading)}$$

The **load impedance Z** can be modeled by a **25.98-Ω resistor in series** with **a capacitor**

$X_C = -15 = -\frac{1}{\omega C}$ → $C = \frac{1}{15\omega} = \frac{1}{15 \times 100\pi} = 212.2\ \mu\text{F}$

Complex Power

- The **complex power S absorbed** by the ac load is the **product of the voltage and the complex conjugate of the current**, or

$$\mathbf{S} = \frac{1}{2}\mathbf{V}\mathbf{I}^* = \mathbf{V}_{\text{rms}}\mathbf{I}^*_{\text{rms}}$$

Where,

$$\mathbf{V}_{\text{rms}} = \frac{\mathbf{V}}{\sqrt{2}} = V_{\text{rms}}\angle\theta_v \qquad \mathbf{I}_{\text{rms}} = \frac{\mathbf{I}}{\sqrt{2}} = I_{\text{rms}}\angle\theta_i$$

$$\mathbf{S} = V_{\text{rms}} I_{\text{rms}} \angle\theta_v - \theta_i$$
$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) + j V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

Complex Power

- The complex power may be expressed in terms of the load impedance Z.

$$\mathbf{Z} = \frac{\mathbf{V}}{\mathbf{I}} = \frac{\mathbf{V}_{\text{rms}}}{\mathbf{I}_{\text{rms}}} = \frac{V_{\text{rms}}}{I_{\text{rms}}} \angle \theta_v - \theta_i$$

Thus, $\mathbf{V}_{\text{rms}} = \mathbf{Z}\mathbf{I}_{\text{rms}}$. Substituting this into above equation

$$\mathbf{S} = I_{\text{rms}}^2 \mathbf{Z} = \frac{V_{\text{rms}}^2}{\mathbf{Z}^*} = \mathbf{V}_{\text{rms}}\mathbf{I}_{\text{rms}}^*$$

Since $Z = R + jX$,

$$\mathbf{S} = I_{\text{rms}}^2 (R + jX) = P + jQ$$

Complex Power

- Where **P and Q** are the **real and imaginary** parts of the complex power; that is,

$$P = \mathrm{Re}(\mathbf{S}) = I_{\mathrm{rms}}^2 R$$
$$Q = \mathrm{Im}(\mathbf{S}) = I_{\mathrm{rms}}^2 X$$

- ➢ **P is the average or real power** and it depends on the load's resistance R.
- ➢ **Q depends on the load's reactance** X and is called the **reactive power**.

Complex Power

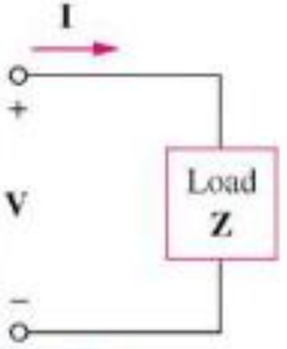
$$\mathbf{S} = I_{\text{rms}}^2(R + jX) = P + jQ$$

$$\mathbf{S} = V_{\text{rms}} I_{\text{rms}} \underline{/\theta_v - \theta_i}$$

$$= V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i) + j V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\theta_v - \theta_i), \qquad Q = V_{\text{rms}} I_{\text{rms}} \sin(\theta_v - \theta_i)$$

Complex Power[Note]



$$S = \frac{1}{2} V I^* = V_{rms} I_{rms} \angle \theta_v - \theta_i$$

$$\Rightarrow S = \underbrace{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}_{P} + j \underbrace{V_{rms} I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

$$S = P + jQ$$

P: is the average power in watts delivered to a load and it is the only useful power.

Q: is the reactive power exchange between the source and the reactive part of the load. It is measured in VAR.

- $Q = 0$ for *resistive loads* (unity pf).
- $Q < 0$ for *capacitive loads* (leading pf).
- $Q > 0$ for *inductive loads* (lagging pf).

The *complex power* in rectangular form is

$$\mathbf{S} = \underbrace{\frac{I_m V_m}{2}\cos(\theta_V - \theta_I)}_{P} + j\underbrace{\frac{I_m V_m}{2}\sin(\theta_V - \theta_I)}_{Q}$$

or

$$\mathbf{S} = P + jQ$$

real or *average power* (P) — *reactive power* (Q)

Units **S**: VA, P: W, Q: VAR

VA = Volt-Amp; VAR = Volt-Amp Reactive

The *impedance* of the element can be expressed as

$$\mathbf{Z}(\omega)=\frac{\mathbf{V}(\omega)}{\mathbf{I}(\omega)}=\frac{V_m\angle\theta_V}{I_m\angle\theta_I}=\frac{V_m}{I_m}\angle(\theta_V-\theta_I)$$

In rectangular form

$$\mathbf{Z}(\omega)=\underbrace{\frac{V_m}{I_m}\cos(\theta_V-\theta_I)}_{R}+j\underbrace{\frac{V_m}{I_m}\sin(\theta_V-\theta_I)}_{X}$$

or

$$\mathbf{Z}(\omega)=R+jX$$

R → resistance, X → reactance

Complex Power[Note]

- The real power P is the average power in watts delivered to a load;
  - *It is the only useful power.*
  - *It is the actual power dissipated by the load.*
- The **reactive power** Q is a measure of the energy exchange between the source and the reactive part of the load.
  - *The unit of Q is the volt-ampere reactive (VAR) to distinguish it from the real power, whose unit is the watt.*

Summary

$$\text{Complex Power} = \mathbf{S} = P + jQ = \mathbf{V}_{\text{rms}}(\mathbf{I}_{\text{rms}})^*$$
$$= |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| \angle \theta_v - \theta_i$$

$$\text{Apparent Power} = S = |\mathbf{S}| = |\mathbf{V}_{\text{rms}}|\,|\mathbf{I}_{\text{rms}}| = \sqrt{P^2 + Q^2}$$

$$\text{Real Power} = P = \text{Re}(\mathbf{S}) = S\cos(\theta_v - \theta_i)$$

$$\text{Reactive Power} = Q = \text{Im}(\mathbf{S}) = S\sin(\theta_v - \theta_i)$$

$$\text{Power Factor} = \frac{P}{S} = \cos(\theta_v - \theta_i)$$

- **Real Power** is the actual power dissipated by the load.
- **Reactive Power** is a measure of the energy exchange between source and reactive part of the load

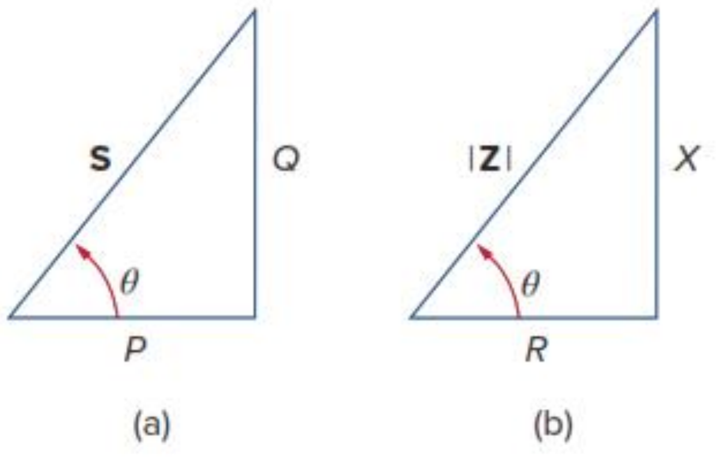
Summary

1. Q = 0 for resistive loads (unity pf).
2. Q < 0 for capacitive loads (leading pf).
3. Q > 0 for inductive loads (lagging pf).

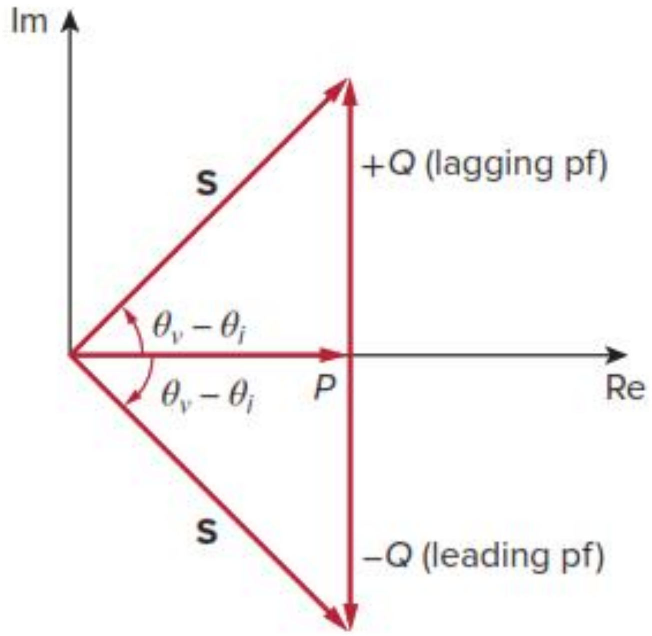
Thus, Complex power (in VA) is the product of the rms voltage phasor and the complex conjugate of the rms current phasor. As a complex quantity, its real part is real power P and its imaginary part is reactive power Q.

Summary

- It is a standard practice to represent **S, P, and Q** in the form of a triangle, known as the power triangle,



(a) Power triangle, (b) impedance triangle.



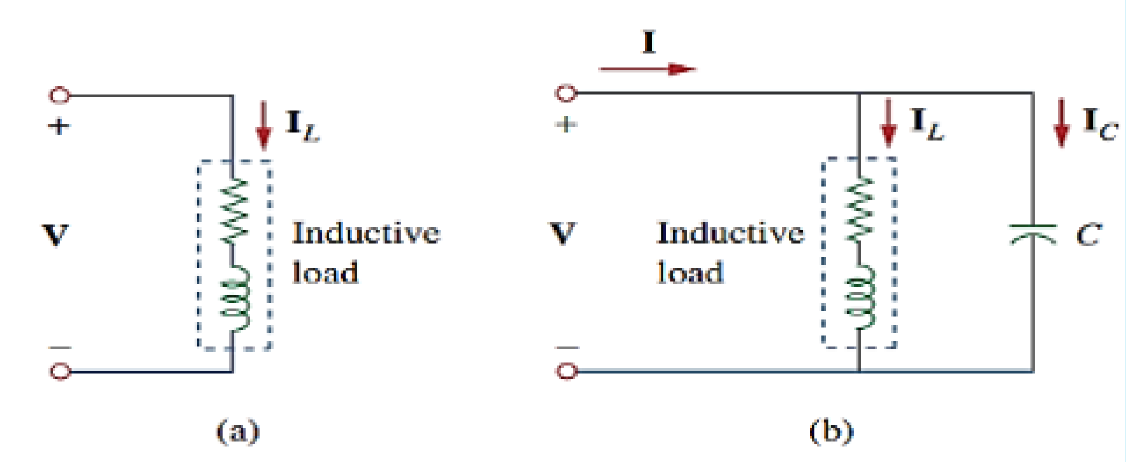
Homework

The voltage across a load is v(t)=60 cos (ωt - 10°) V and the current through the element in the direction of the voltage drop is i(t)=1.5 cos (ωt + 50°) A. Find: (a) the complex and apparent powers, (b) the real and reactive powers, and (c) the power factor and the load impedance.

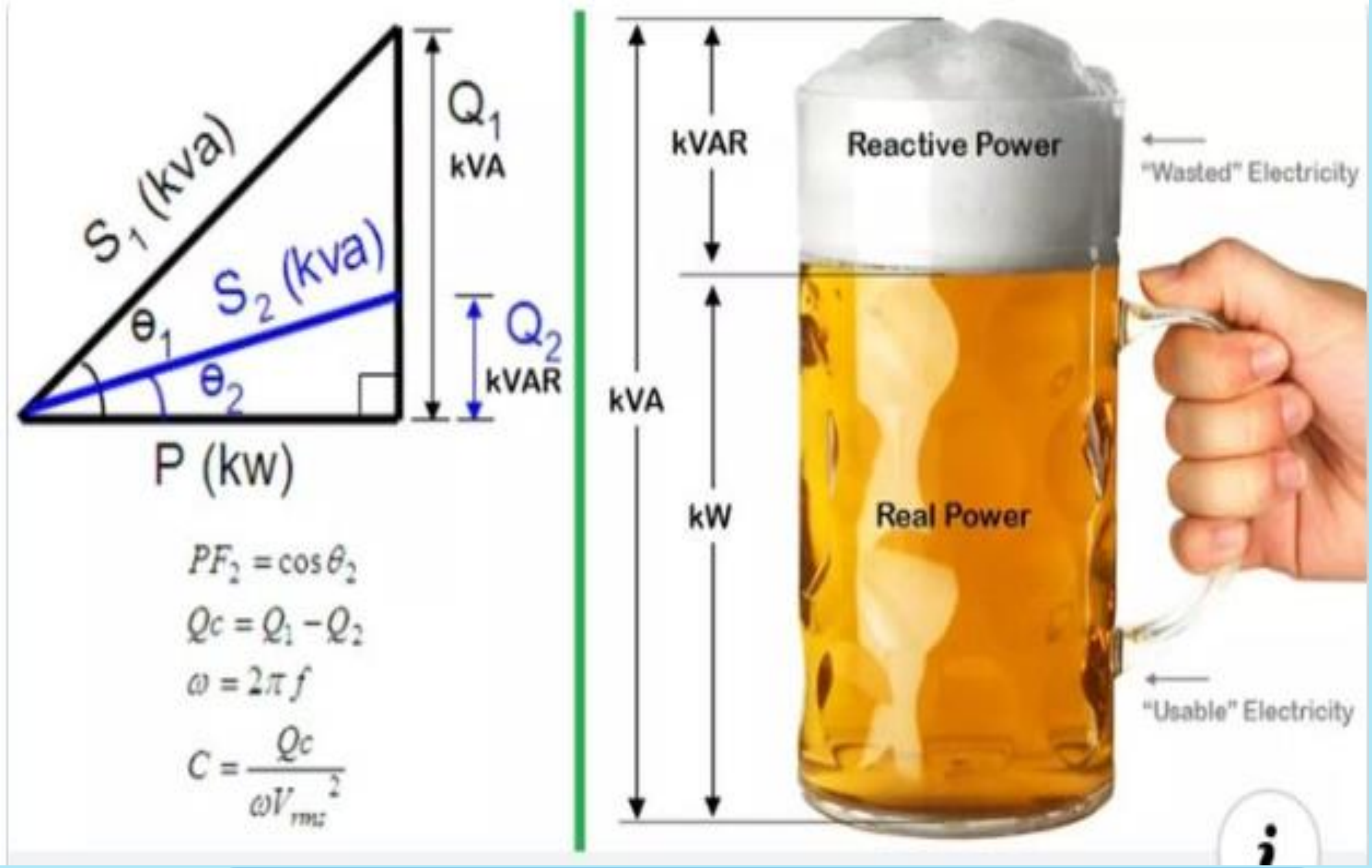
ANS: a). 45 (60°) VA b). $P = 22.5$ W, $Q = -38.97$ VAR c). 0.5 (leading)

Power factor and power factor correction

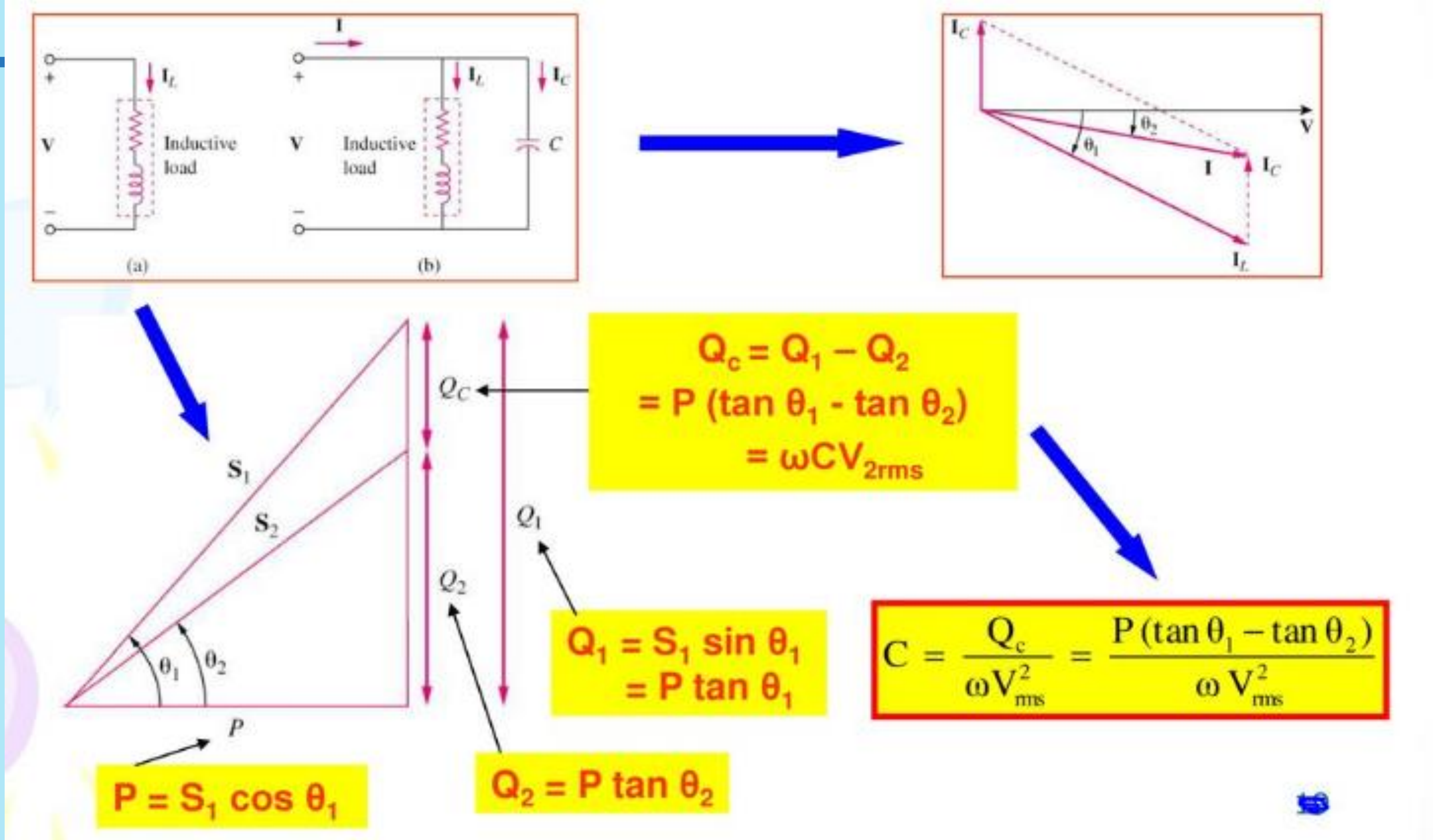
- The process of increasing the power factor without changing the voltage or current to the original load is **known as power factor correction**.
- Since most loads are inductive, as shown in Fig.(a)**, a load's power factor is improved or corrected by deliberately installing a capacitor in parallel with the load,** as shown in Fig. (b).



PF


Q1
kVA
S1 (kva)
S2 (kva)
θ1
θ2
Q2
kVAR
P (kw)
PF2 = cos θ2
Qc = Q1 − Q2
ω = 2πf
C = Qc / ωV_rms^2
kVAR
Reactive Power
"Wasted" Electricity
kVA
kW
Real Power
"Usable" Electricity

Power factor and power factor correction



Power factor and power factor correction

- The reduction in the **reactive power is caused by the shunt capacitor**; that is,

$$Q_C = Q_1 - Q_2 = P(\tan\theta_1 - \tan\theta_2) \quad Q_C = V_{\text{rms}}^2/X_C = \omega C V_{\text{rms}}^2.$$

- The value of the required shunt capacitance C is determined as

$$C = \frac{Q_C}{\omega V_{\text{rms}}^2} = \frac{P(\tan\theta_1 - \tan\theta_2)}{\omega V_{\text{rms}}^2}$$

Power factor and power factor correction

- **Shunt inductance L** can be calculated from

$$Q_L = \frac{V_{rms}^2}{X_L} = \frac{V_{rms}^2}{\omega L} \quad \Rightarrow \quad L = \frac{V_{rms}^2}{\omega Q_L}$$

- Where QL = Q1 - Q2, the difference between the new and old reactive powers.

Example

When connected to a 120-V (rms), 60-Hz power line, a load absorbs 4 kW at a lagging power factor of **0.8.** Find the **value of capacitance** necessary to raise the pf to **0.95**.

Solution

- If the pf = 0.8, then

$\cos \theta 1 = 0.8 \Rightarrow \theta 1 = 36.87°$

Where

θ1 is the phase difference between voltage and current. We obtain the apparent power from the real power and the pf as

$$S_1 = \frac{P}{\cos \theta_1} = \frac{4000}{0.8} = 5000 \text{ VA}$$

The reactive power is

Q1 = S1 sin θ = 5000 sin 36.87 = 3000 VAR

Solution

- When the **pf is raised to 0.95,**

$$\cos \theta_2 = 0.95 \Rightarrow \theta_2 = 18.19°$$

- The **real power P** has not changed. But the apparent power has changed; its new value is

$$S_2 = \frac{P}{\cos \theta_2} = \frac{4000}{0.95} = 4210.5 \text{ VA}$$

- The new reactive power is **Q2 = S2 sin θ2 = 1314.4 VAR**

Solution

- The difference between the new and old reactive powers is due to the parallel addition of the capacitor to the load. The reactive power due to the capacitor is

QC = Q1 – Q2 = 3000 – 1314.4 = 1685.6 VAR

$$C = \frac{Q_C}{\omega V_{rms}^2} = \frac{1685.6}{2\pi \times 60 \times 120^2} = 310.5\ \mu F$$

Benefits of reading